

**Supplementary Table 1. Collinearity analysis of each factor**

| Variables                           | VIF  | VIF_CI_low | VIF_CI_high |
|-------------------------------------|------|------------|-------------|
| Age (years)                         | 1.21 | 1.07       | 1.64        |
| Gender (male)                       | 1.04 | 1.00       | 4.05        |
| BMI (kg/m <sup>2</sup> )            | 1.30 | 1.12       | 1.72        |
| Smoking (yes)                       | 1.15 | 1.03       | 1.62        |
| ASA classification (III-IV)         | 1.15 | 1.03       | 1.62        |
| Proximal humeral fractures (yes)    | 1.10 | 1.01       | 1.72        |
| Open fracture (yes)                 | 1.16 | 1.04       | 1.62        |
| High-energy injury (yes)            | 1.07 | 1.01       | 2.00        |
| History of diabetes (yes)           | 2.46 | 1.94       | 3.27        |
| Duration of surgery (minutes)       | 1.14 | 1.03       | 1.63        |
| Debridement times ( $\geq 2$ times) | 1.16 | 1.04       | 1.62        |
| Recovery initiation time (weeks)    | 1.74 | 1.43       | 2.28        |
| HbA1c (%)                           | 3.08 | 2.38       | 4.12        |

Note: The data in the table are the variance inflation factor (VIF) and its 95% confidence interval (CI), used to assess the degree of collinearity among the variables. VIF values are all  $< 5$ , suggesting no significant collinearity among the variables.

**Supplementary Table 2. Multivariate regression and interaction of HbA1c as a categorical variable (sensitivity analysis)**

| Variables          | Model 3             |         | Model 4             |         | Model 5             |         |
|--------------------|---------------------|---------|---------------------|---------|---------------------|---------|
| Main effect        | $\beta$ (95% CI)    | P value | $\beta$ (95% CI)    | P value | $\beta$ (95% CI)    | P value |
| HbA1c 6.5% to 8.0% | 5.87 (-2.34, 14.08) | 0.159   | 4.21 (-6.12, 14.54) | 0.421   | 5.63 (-3.89, 15.15) | 0.243   |
| HbA1c $> 8.0\%$    | 10.23 (0.87, 19.59) | 0.033   | 8.45 (-3.21, 20.11) | 0.153   | 9.87 (-0.98, 20.72) | 0.074   |

|                                                       |                       |       |                       |       |                       |       |
|-------------------------------------------------------|-----------------------|-------|-----------------------|-------|-----------------------|-------|
| Recovery initiation time (weeks)                      | 3.12 (1.08, 5.16)     | 0.003 | 2.01 (-1.23, 5.25)    | 0.221 | 3.08 (1.02, 5.14)     | 0.004 |
| Debridement $\geq 2$ times                            | 4.87 (-2.01, 11.75)   | 0.163 | 4.76 (-2.15, 11.67)   | 0.175 | 3.21 (-8.76, 15.18)   | 0.597 |
| Interaction items                                     |                       |       |                       |       |                       |       |
| HbA1c (6.5-8) $\times$ recovery time                  | -                     | -     | 1.87 (-2.34, 6.08)    | 0.381 | -                     | -     |
| HbA1c ( $> 8.0$ ) $\times$ recovery time              | -                     | -     | 4.23 (-0.98, 9.44)    | 0.111 | -                     | -     |
| HbA1c (6.5 to 8) $\times$ debridement $\geq 2$ times  | -                     | -     | -                     | -     | 1.54 (-11.23, 14.31)  | 0.812 |
| HbA1c ( $> 8.0$ ) $\times$ debridement $\geq 2$ times | -                     | -     | -                     | -     | 3.87 (-12.34, 20.08)  | 0.637 |
| Covariate                                             | Adjusted <sup>1</sup> | -     | Adjusted <sup>1</sup> | -     | Adjusted <sup>1</sup> | -     |
| Adjust R <sup>2</sup>                                 | 0.415                 | -     | 0.421                 | -     | 0.409                 | -     |

Note: With HbA1c as the categorical variable ( $< 6.5\%$  as the reference group), analyze its main effect and interaction with rehabilitation initiation time and debridement frequency. Covariates such as age, gender, BMI, smoking, ASA classification, fracture type, high-energy injury, history of diabetes, and duration of surgery were adjusted for all models. "-" indicates that this item was not included in the model.

**Supplementary Table 3. Generalized Additive Model (GAM) validates the linear relationship between HbA1c and DASH score**

| Categories      | Variables      | edf  | F value | Estimates | Standard error | P value  |
|-----------------|----------------|------|---------|-----------|----------------|----------|
| Smoothing terms | s(HbA1c)       | 1.00 | 50.77   | —         | —              | $<0.001$ |
| Linear term     | Age            | —    | —       | -0.04     | 0.12           | 0.744    |
|                 | BMI            | —    | —       | -0.18     | 0.36           | 0.605    |
|                 | Open fractures | —    | —       | 3.48      | 3.56           | 0.331    |

Note: The degree of freedom (edf) of the smoothing term  $s(\text{HbA1c})$  is 1.00, suggesting a significant linear association between the two ( $P < 0.001$ ); Linear terms are covariates included in the model, and "-" indicates no corresponding statistical value.