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Intra-racial Variations in the Profile Pattern of the Human Facial Skeleton.¹

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I. Introduction.

The interest in the intra-racial variations of the facial skeleton has increased considerably after it was shown that different types of malocclusions are associated with certain facial forms (R. SCHWARZ, 1926; OPPENHEIM, 1927; HELLMAN, 1931; CIE-SZYNSKI, 1933, et al.). Since most occlusion anomalies are reflected in the profile pattern of the face, the studies have been concentrated on the form of the cranium seen from this norma. During recent years more and more attempts have been made to disclose correlations between different parts of the viscerocranium (MORANT, 1936; BJÖRK, 1947; BUSHRA, 1948, and others). By means of the so-called tres indices principle (see FÜRST, 1933) the interrelation both of the various dimensions of the viscerocranium and of the dimensions of the viscerocranium and the neurocranium has been further studied (HJORTSJÖ, 1947). Up to the present time, however, no systematic study has been carried out of the intra-facial variations in norma lateralis.

In the present work the facial skeleton has been studied with the aid of two triangular profile diagrams, one for the upper face and one for the lower jaw. The aim has been to create a uniform picture of the normal intra-racial variations of the facial skeleton by combining two investigations: 1. an analysis of the

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conditions of variability of the different components of the profile diagrams, 2. a systematic analysis of the interrelation of the components.

II. Methods.

A. Methods of Measurement.

The aim of this study has been, among other things, to compare the shape and size of the profile pattern of the human facial skeleton, expressed in angular and linear measurements. In previous studies of the profile pattern of the facial skeleton polygonal diagrams were used to represent the facial structure. These diagrams were constructed by connecting certain defined points of measurement. Similar profile diagrams, constructed in the same way, have also been used in this investigation. A profile diagram with the simplest geometric structure was sought in order to simplify the statistical calculations.

Within the field of anthropology the so-called *facial triangle* has long been used in the study of the viscerocranium in the norma lateralis, and it forms the basis of the present investigation. According to MARTIN (1928) its sides are: the cranial base length (Nasion-Basion length), the superior facial length (Basion-Prosthion length), and the superior facial height (Nasion-Prosthion height). In the following the sides of the triangle will be represented by: Na-Ba (the cranial base length), Ba-Pr (the superior facial length), Na-Pr (the superior facial height). The angles of the triangle will be represented as follows: Na (the angle Basion-Nasion-Prosthion), Pr (the angle Nasion-Prosthion-Basion) and Ba (the angle Prosthion-Basion-Nasion). See figure 1 A.

KIEFFER (1908) points out that every mandible, regardless of growth or atrophy, in the profile pattern forms a triangle in which the corners are formed by the point of the chin, the condyle, and the dorso-caudal portion of the angulus mandibulae. Since the corner points in this triangle are not situated in the median plane, as in the facial triangle, the mandibular diagram must be constructed by projecting the profile pattern into the median plane. This is easily done by means of a graphic construction. As the points of measurement are difficult to determine, however, the method has not been used in this study. Instead, the study of the mandible has been based on a triangular diagram, constructed in the following manner. The lower jaw was

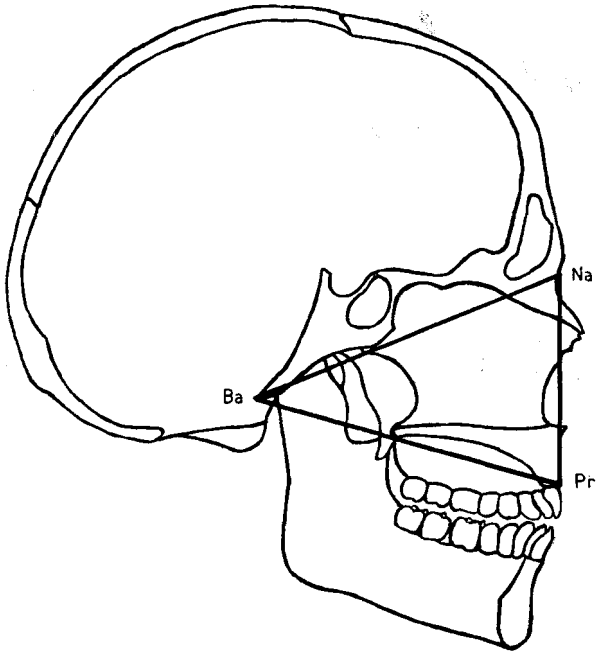


Fig. 1 A. The facial triangle traced in the median plane of the skull. Na = Nasion, Pr = Prosthion, Ba = Basion.

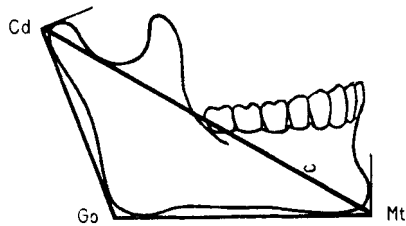


Fig. 1 B. The mandibular triangle traced in the median plane of the skull. Cd = Condylion medianum, Go = Gonion medianum, Mt = Mentale.

placed in a so-called mandibulometer (figure 2) so that the corpus mandibulae stands on the horizontal plane, A, and the two rami ascendentes are touched by plane B, partly on the back of the capitulum and partly on the dorsal edge of the angulus area; when the structure of the mandible is asymmetrical the plane B is allowed to touch the left ramus ascendens in the manner described above and the right ramus either on the back of the capitulum or on the dorsal edge of the angulus portion, dependent

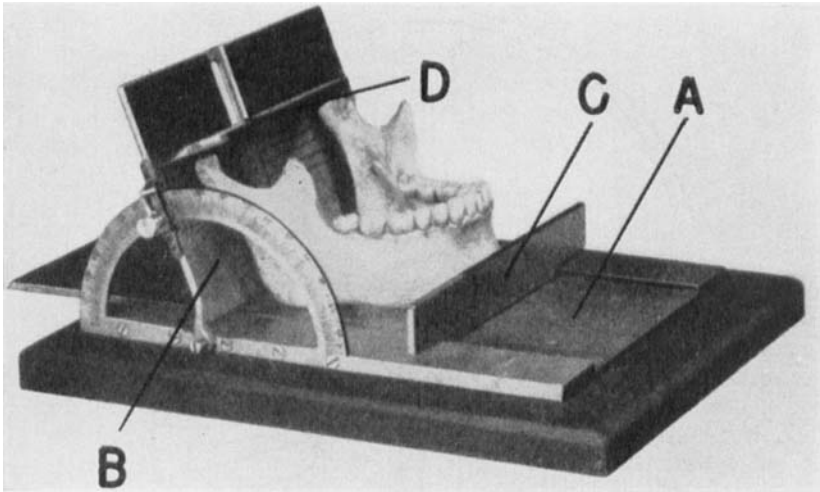


Fig. 2. Mandibulometer (after MARTIN, 1928).

on which of these parts is first touched by plane B when the lower jaw is moved towards this plane. Planes C and D which are perpendicular to the sagittal plane and to the planes A and B, respectively, are moved towards the mandible. The plane C touches the most prominent point of the chin section. The plane D touches the top of the capitulum on the left ramus ascendens. The angle between the planes A and B is measured, as well as the perpendicular distances from the section line between these planes to the section lines between the planes A and C, and B and D. The values obtained represent in the order mentioned: the jaw angle, the length of the corpus, and the height of the ramus (the method of measurement agrees with that recommended by MORANT, 1936). The magnitudes obtained are measured in the sagittal plane and they are, therefore, identical with their projections towards the median plane. The above-mentioned measuring points are all situated outside the profile picture of the lower jaw, being formed by the section lines between the various tangents. In spite of this the measuring points in this paper have been designated according to the adjacent part of the mandible. Thus, the section point between the planes A and C which are projected towards the median plane is called Mentale (Mt), the section point between the projected planes A and B Gonion medianum (Go), and the section point between the projected planes

B and D Condylion medianum (Cd), see figure 1 B. The denominations Gonion medianum and Condylion medianum have been introduced in agreement with the terminology used by EDLÉN, HJORTSJÖ, LINDH (1947) to represent the projection of symmetrical points to the median plane.

The profile diagram of the lower jaw is completed by joining the points Cd and Mt in the median plane. The resulting diagram will subsequently be called the *mandibular triangle*. The third side of the triangle, Cd-Mt, here called the condylomental line, is obtained, as is the case of the angles Mentale-Condylion medianum-Gonion medianum (called Cd) and Gonion medianum-Mentale-Condylion medianum (called Mt), from the values obtained with the mandibulometer by means of calculation, from the cosinus and sinus theorems, or by graphic construction. In the latter case the trigonometer constructed by FÜRST (1906) is recommended.

B. Material.

Since, due to random choice, biometrical investigations may give varying results when undertaken with different samples, this report is based on cranial material from two different races. Only male skulls have been included so as to eliminate any influence of sexual differences on the results.

One series, the Oslo material, comprises 150 crania and the other, the Lapp material, 176 crania. The Oslo material forms the largest group in SCHREINER's work *Crania Norvegica* (1939). The crania in this series are mostly dolichocephalous and are taken from medieval graves in Oslo. The Lapp material comes from present-day graves in the Norwegian Finnmark; the crania in this series are mostly brachycephalous (SCHREINER: *Zur Osteologie der Lappen* (1935)).

Both materials are considerably smaller in this work than in SCHREINER's as only crania with lower jaws are included.

C. Statistical Methods.

The numerical values which in the following express the magnitude of the sides and angles in the facial diagrams are given in millimeters and degrees, respectively.

In agreement with MORANT (1936) and other authors, the variability of the various facial measures is given in the standard

deviation for angular measurements and in the coefficient of variation (= the standard deviation in per cent of the mean) for linear measures.

Analysis of the relations within and between the facial and mandibular triangles is carried out as follows. The 12 components of the triangles (3 sides and 3 angles in each triangle) are studied systematically regarding their correlation to each other. One after the other, the 12 components are selected as the independent variable. As the value of this variable is changed the simultaneous variation of the other 11 components are registered.

The change in value of the independent variable is carried out as follows. Starting from the mean value of the independent variable in the material, the latter is divided into two groups. Crania belonging to Group I have values for the independent variable which are below the mean value of the variable in the material as a whole. Similarly, Group II comprises crania whose values for the independent variable exceed the mean value of the variable in the total material. The mean of the independent variable is then calculated for the two groups, as well as the difference between these mean values.

Thus two partial materials are created (the groups I and II), with different mean values for the independent variable. The degree of co-variation of the other components of the triangle with the independent variable is determined in the following manner. In both groups the mean values of the 11 dependent variables are calculated and thereafter the differences between the corresponding values in the two groups. If there is a statistically ascertained difference between these figures, a correlation exists between the dependent variable in question and the independent variable.

In accordance with the principle of the division of the material into groups I and II, the mean value for the independent variable is always greater in group II than in group I. If, therefore, in those cases where a difference in the means for a dependent variable in groups I and II has been demonstrated, the mean value for the dependent variable is greatest in group II, a *positive* correlation exists; when, on the other hand, the value for the dependent variable is greatest in group I there is a *negative* correlation.

In figures 3 to 14 the mean values in group I are represented by the unbroken lines and those in group II by the broken lines

for the 12 components of the triangle (the independent as well as the dependent variables). Due to the lack of space it is not, however, possible to publish the numerical values.

The differences between the mean values in groups I and II ($D_{M_{II}-M_I}$) are given in tables 2 to 17. These tables also show the degree of statistical certainty for the differences obtained, expressed by the quotient of the difference and its mean error

$$\left(\frac{D_{M_{II}-M_I}}{\epsilon_{DM_{II}-M_I}} \right).$$

The obtained difference is accepted as significant if the quotient exceeds 3.00 and considered probable if the quotient lies between 2.50—3.00.

Space permits only a discussion of the qualitative correlation analysis in this paper. As regards the quantitative analysis the reader is referred to tables 2 to 17, in which the relative difference between the groups I and II (*i. e.*, the difference in per cent of the mean value in group I) has been inserted for linear measurements.

III. Conditions of Variability within the Facial and Mandibular Triangles (see tables 1 A and 1 B).

In both materials, in the facial triangle, the cranial base length, Na-Ba, is the side that has the lowest coefficient of variation. The greatest variability is in both materials shown by the side Na-Pr.

In the mandibular triangle the condylomental line, Cd-Mt, has the lowest variability and the ramus height the highest. Here, too, the findings from both materials are in agreement.

Within the facial triangle as well as within the mandibular triangle there is a tendency towards negative correlation between the coefficient of variation and the size of the sides of the triangle (cf. MORANT, 1936).

The angles of the facial triangle seem to have about the same standard deviation in the two materials. Of the angles of the mandibular triangle, Go has the largest standard deviation and Mt the lowest.

It is remarkable that the angle with the highest variability in the mandibular triangle (Go) is opposite to the side with the

Table 1 A.

Means and coefficients of variation for the linear measurements of the facial and mandibular triangles.

	Oslo material		Lapp material	
	Mean measurement.	Coeff. of variat.	Mean measurement.	Coeff. of variat.
Na-Pr	72.8 mm	5.6	67.7 mm	6.2
Ba-Pr	97.5 »	5.1	97.0 »	5.5
Na-Ba	102.4 »	4.3	98.6 »	3.9
Cd-Go	65.5 »	7.2	58.2 »	9.5
Go-Mt	77.0 »	5.6	72.3 »	6.7
Cd-Mt	121.6 »	4.5	116.4 »	5.9

Table 1 B.

Means and standard deviations for the angular measurements of the facial and mandibular triangles.

	Oslo material		Lapp material	
	Mean measurement.	Stand. dev.	Mean measurement.	Stand. dev.
Na	64.6 °	3.7	68.8 °	3.2
Pr	72.2 °	3.1	70.4 °	3.2
Ba	42.9 °	3.1	40.6 °	2.6
Cd	34.2 °	3.7	31.7 °	4.5
Go	116.6 °	6.1	122.3 °	7.5
Mt	27.8 °	3.3	23.8 °	3.9

lowest variability (Cd-Mt), and that the angle with the lowest variability (Mt) is opposite to the side with the highest variability (Cd-Go).

IV. Analysis of the Relations within and between the Facial and Mandibular Triangles.

A. Relations within the Facial Triangle.

The cranial base length, Na-Ba, shows in both materials a pronounced positive correlation with the superior facial length, Ba-Pr (table 2, figures 4 and 5). In the Lapp material there is, moreover, a probable positive correlation with the superior facial height, Na-Pr; in the Oslo material this correlation is not prob-

able (table 2, figures 3 and 5). The correlation between Na-Pr and Ba-Pr is significantly positive in the Lapp material but is non-existent in the Oslo material (table 2, figures 3 and 4).

Between the angle Na and the other angles of the facial triangle, Pr and Ba, there is a significant negative correlation in both materials (table 3, figures 6, 7, and 8). An analysis of the relation between the angles Pr and Ba shows that there is a significant negative correlation between them in the Oslo material but this correlation is not probable in the Lapp material (table 3, figures 7 and 8).

The cranial base length, Na-Ba, is the side in the facial triangle that shows the lowest correlation to the angles of the triangle. Na-Ba has no correlation to the angle Na in either material (tables 4 and 5, figures 5 and 6), but has a significant negative correlation to the adjacent angle, Ba (tables 4 and 5, figures 5 and 8). Between Na-Ba and the third angle, Pr, there exists a significant positive correlation in the Oslo material; in the Lapp material it is not even probable (tables 4 and 5, figures 5 and 7).

In both materials a significant positive correlation is found between the side Ba-Pr and the opposite angle, Na (tables 4 and 5, figures 4 and 6). There is a significant negative correlation between Ba-Pr and the angle Ba in the Oslo material while in the Lapp material the correlation is barely significant (tables 4 and 5, figures 4 and 8). The Lapp material shows a significant negative correlation between Ba-Pr and the third angle, Pr; in the Oslo material the correlation is not even probable (tables 4 and 5, figures 4 and 7).

There is in both materials a significant positive correlation between the side Na-Pr and the opposite angle, Ba (tables 4 and 5, figures 3 and 8). In the Lapp material there is also a significant negative correlation between Na-Pr and the angle Pr; in the Oslo material the correlation is probable (tables 4 and 5; figures 3 and 7). Na-Pr has a significant negative correlation to the third angle, Na, in the Oslo material; in the Lapp material, on the other hand, co-variability is not probable (tables 4 and 5, figures 3 and 6).

B. Relations within the Mandibular Triangle.

Between the condylomental line, Cd-Mt, and the ramus height, Cd-Go, as well as the corpus length, Go-Mt, there is a significant

positive correlation in both materials (table 6, figures 9, 10 and 11). Cd-Go and Go-Mt in the Oslo material show a probable positive correlation to each other; in the Lapp material this correlation owns a probability almost amounting to significance (table 6, figures 9 and 10).

The angles of the mandibular triangle exhibit a very strong co-variability. There is in both materials a significant negative correlation between the angles Go and Cd, and between the angles Go and Mt (table 7, figures 12, 13, and 14). The two acute angles, Cd and Mt, show in both materials a significant positive correlation (table 7, figures 12 and 14).

The ramus height, Cd-Go, has in both materials a significant negative correlation to the jaw angle, Go (tables 8 and 9, figures 9 and 13), and a significant positive correlation to the opposite angle, Mt (tables 8 and 9, figures 9 and 14). Between Cd-Go and the remaining angle, Cd, however, the correlation is not probable in either material (tables 8 and 9, figures 9 and 12).

In both materials the corpus length, Go-Mt, stands in a significant positive correlation to the opposite angle, Cd, but is not correlated to the adjacent angle, Mt (tables 8 and 9, figures 10, 12, and 14). There is in the Oslo material a negative correlation bordering on significance between Go-Mt and the third angle, Go; a similar tendency can be traced in the Lapp material although the conditions for probability are not fulfilled (tables 8 and 9, figures 10 and 13).

Of the sides of the mandibular triangle the lowest correlation to the angles of the triangle is shown by the condylomental line, Cd-Mt. Thus there is no correlation in either material between Cd-Mt and the angle Mt; between the same side and the angle Cd there is a probable negative correlation in the Oslo material, while in the Lapp material there is no correlation (tables 8 and 9, figures 11, 12, and 14). The connexion between Cd-Mt and the angle Go appears differently in both materials, dependent on which of the two components is selected as the independent variable. When Cd-Mt is the independent variable, Go shows no tendency of co-variability (table 8, figure 11). If, on the other hand, Go is taken as the independent variable, a probable positive co-variability for Cd-Mt is found in the Oslo material and a tendency in this direction is noted in the Lapp material (table 9, figure 13).

C. Relations between the Facial and Mandibular Triangles.

The Lapp material presents a significant positive correlation between the superior facial height, Na-Pr, and the two sides of the mandible, Go-Mt and Cd-Mt. There is also, in this material, a positive correlation, bordering on significance, between Na-Pr and Cd-Go. In the Oslo material, however, the correlation between Na-Pr and the three sides of the mandibular triangle is not probable (tables 10 and 11, figures 3, 9, 10, and 11).

The superior facial length, Ba-Pr, has in the Lapp material not even a probable correlation to any of the sides of the mandibular triangle while in the Oslo material a probable positive correlation exists between Ba-Pr and Cd-Mt; otherwise there is no correlation to the sides of the mandibular triangle in this material either (tables 10 and 11, figures 4, 9, 10, and 11).

The cranial base length, Na-Ba, has in the Oslo material a significant positive correlation to Cd-Mt. The same material shows a probable positive correlation between Na-Ba and Go-Mt and a positive correlation bordering on significance between Na-Ba and Cd-Go. The correlation between Na-Ba and Go-Mt, and Na-Ba and Cd-Go in the Lapp material is not probable. There is, however, a positive correlation bordering on significance between the sides Na-Ba and Cd-Mt (tables 10 and 11, figures 5, 9, 10, and 11).

The greatest co-variant with the sides of the mandibular triangle is thus in the Oslo material Na-Ba and in the Lapp material Na-Pr. In both materials the mandibular side, Cd-Mt, shows the highest correlation to the sides of the facial triangle.

There is no correlation at all between the angles of the facial triangle and the angles of the mandibular triangle in the Lapp material. In the Oslo material there is a probable negative correlation between angles Pr and Go. Otherwise there is no correlation between the angles of the facial and mandibular triangles (tables 12 and 13, figures 6, 7, 8, 12, 13, and 14).

The sides of the facial triangle have no correlation to the angles of the mandibular triangle in either material (tables 14 and 15, figures 3, 4, 5, 12, 13, and 14).

No correlation exists in the Oslo material between the sides of the mandibular triangle and the angles of the facial triangle. The same is the case for the Lapp material, with the exception

of a significant positive correlation between Cd-Mt and the angle Ba (tables 16 and 17, figures 6, 7, 8, 9, 10, and 11).

V. Discussion.

The correlations found between the sides and angles within the facial and mandibular triangles indicate the existence of a connexion between shape and size, represented here by angular and linear measurements, respectively. It can be demonstrated, however, that certain sides of the triangles show a closer connexion with the shape of the triangle, *i. e.*, with its angles, than others do. Similarly, certain angles show higher correlation to the size of the triangle, *i. e.*, to its sides, than others.

Thus, within the facial triangle, the cranial base length, Na-Ba, exhibits a lower degree of co-variability with the angles of the triangle than the other two sides, Ba-Pr and Na-Pr. The cranial base length, has therefore, the least connexion of the three sides of the facial triangle with the shape of the triangle. The angle Ba, which shows a significant correlation to all the sides of the facial triangle, has the greatest connexion with the size of the triangle.

Certain relations, analogous to those within the facial triangle, can be traced within the mandibular triangle. One of the sides of the triangle, the condylomental line, Cd-Mt, shows a lesser degree of co-variability with the shape of the triangle than the other two sides. The angle Go seems to have the highest correlation to the size of the triangle.

Studies on the variability of the different components of the triangles indicate that the cranial base length, Na-Ba, and the condylomental line, Cd-Mt, are the sides of the facial and mandibular triangles with the lowest variability. The reason is, at least partly, the negative correlation which has been demonstrated to exist between the other two sides of the triangles and the angle lying between them.

An analysis of the relations between the components of the facial and mandibular triangles reveals that, with the above-mentioned exceptions, there is no correlation either between the size (*i. e.*, the sides) of one triangle and the shape (*i. e.*, the angles) of the other or between the shapes (*i. e.*, the angles) of the two triangles. A correlation seems to exist, however, between the sizes (*i. e.*, the sides) of the two triangles. Of the sides of the facial triangle

the cranial base length has the highest correlation to the sides of the mandibular triangle. In this triangle the condylomental line has the highest correlation to the sides of the facial triangle. Of the correlations found between the sides of the two triangles only that between the cranial base length and the condylomental line is significant in *both* materials (bordering on significance in the Lapp material).

VI. Summary.

By means of two triangular diagrams, one for the upper face and one for the lower jaw (see figures 1 A and B) variations within the profile pattern of the facial skeleton were studied. The investigation is based on male crania from two different races, 150 in the Oslo material and 176 in the Lapp material.

The relations between the components of the profile diagrams in both materials are tabulated in table 18. Regarding the denominations for sides and angles the reader is referred to the chapter on "Methods of measurement".

Positive correlations of varying degrees of probability were found to exist between the sides of the facial triangle. Between the angles of this triangle the correlations were negative. The cranial base length, Na-Ba, had the lowest correlation to the angles of the triangle. The angle Ba had the highest correlation to the sides of the triangle.

In the mandibular triangle there were positive correlations between the sides. A negative correlation existed between the obtuse angle, Go, and the two others, which had a mutual positive correlation. The condylomental line, Cd-Mt, was the side with the lowest correlation to the angles of the mandibular triangle and of the angles Go showed the highest correlation to the sides of the triangle.

There was no correlation either between the sides of one triangle and the angles of the other or between the angles of the two triangles — with a few exceptions, see table 18. The sides of the triangles were correlated, however; in both materials a significant value was found only between the cranial base length, Na-Ba, and the condylomental line, Cd-Mt.

In the facial triangle the cranial base length was the side with the lowest variability and the superior facial height, Na-Pr, the side with the highest. The angles of the triangle were approximately equally variable.

Of the sides in the mandibular triangle, the condylomental line, Cd-Mt, had the lowest variability and the ramus height, Cd-Go, the highest. The angle Go had the highest and the angle Mt the lowest variability.

Analogous intra-triangular conditions existed for the cranial base length, Na-Ba, and the condylomental line, Cd-Mt. Both sides were thus fairly independent of the shape, *i. e.*, of the angles of the triangles. These two sides had the lowest variability in both the facial and the mandibular triangles. Finally, these sides were in both materials the only ones with a significant inter-triangular correlation.

Zusammenfassung.

Mit Hilfe von zwei dreieckigen Diagrammen — eins für das Obergesicht und eins für den Unterkiefer (s. Fig. 1 A und B) — studiert der Verf. Variationen im Profilbild des Gesichtsskeletts. Die Untersuchung basiert auf zwei Serien von männlichen Schädeln verschiedener Rassen. Die beiden Materialreihen umfassen 150 bzw. 176 Fälle.

Tabelle 18 bietet eine Zusammenstellung betr. die Relationen zwischen den Komponenten der Profildiagramme aus beiden Schädelmaterialserien. Über die Bezeichnungen der Seiten und Winkel siehe unter »Methods of measurement«.

Zwischen den Seiten des Gesichtsdreiecks bestehen positive Korrelationen wechselnden Wahrscheinlichkeitsgrades, zwischen den Winkeln des gleichen Dreiecks liegen negative Korrelationen vor. Die Schädelbasislänge (Na-Ba) ist diejenige Seite des Dreiecks, die den Dreieckswinkeln am schwächsten korreliert ist. Von den letzteren ist der Winkel Ba den Seiten des Dreiecks am stärksten korreliert.

Im Mandibulardreieck liegen positive Korrelationen zwischen den Seiten vor. Zwischen dem stumpfen Winkel (Go) und den beiden übrigen herrscht negative Korrelation, unter den beiden letztgenannten herrscht positive Korrelation. Von den Seiten ist die Kondylomentallinie (Cd-Mt) den Winkeln des Mandibulardreiecks am schwächsten korreliert, und von den Winkeln ist Go am stärksten mit den Seiten dieses Dreiecks korreliert.

Zwischen den Seiten des einen Dreiecks und den Winkeln des anderen, sowie zwischen den Winkeln der beiden Dreiecke besteht keine Korrelation — bis auf ein paar Ausnahmen, s. Tab. 18. Dagegen weisen die Seiten des einen Dreiecks Korrelation zu den

Seiten des anderen auf; signifikant ist die Korrelation in den untersuchten Materialserien jedoch nur zwischen der Schädelbasislänge (Na-Ba) und der Kondylomentallinie (Cd-Mt).

Im Gesichtsdreieck ist die Schädelbasislänge die am wenigsten variable Seite, die Obergesichtshöhe (Na-Pr) die am stärksten variable. Die Dreieckswinkel sind ungefähr gleich stark variabel.

Im Mandibulardreieck ist die Kondylomentallinie (Cd-Mt) die am wenigsten und die Ramushöhe (Cd-Go) die am stärksten variable Seite. Von den Winkeln dieses Dreiecks ist Go der höchstvariable und Mt der am wenigsten variable Winkel.

Betreffs der Schädelbasislänge (Na-Ba) und der Kondylomentallinie (Cd-Mt) herrschen analoge intratrianguläre Verhältnisse. So sind beide Seiten ziemlich unabhängig von der Form der Dreiecke, d. h. von den Winkeln derselben. Ferner sind diese beiden Seiten im Gesichts- bzw. Mandibulardreieck die am wenigsten variablen Seiten. Schliesslich sind diese Seiten die einzigen, die in beiden Schädelreihen eine signifikante intertrianguläre Korrelation aufweisen.

Résumé.

Avec l'aide de deux diagrammes triangulaires en profil, un pour la partie supérieure de la face, l'autre pour la mâchoire inférieure (voir fig. 1. A et B), les variations du profil de la squellette faciale peuvent être étudiées. La recherche se base sur deux séries de crânes masculins de races différentes. Les matériaux comprennent 150 respectivement 176 cas.

Tableau 18 montre une juxtaposition des matériaux des deux crânes sur les relations entre les éléments des diagrammes cidessus. Concernant les dénominations des cotés et des angles voir fig. 1 A. et B.

Entre les côtés du triangle facial, des corrélations positives d'un degré de probabilité varié existent, entre les angles du même triangle des corrélations négatives. La longueur de la base cranienne (Na--Ba) est celui des côtés du triangle qui est le moins corrélatif avec les angles du triangle. De ceux-ci l'angle Ba est le plus corrélatif avec les côtés du triangle.

Dans le triangle mandibulaire des corrélations positives existent entre les côtés. Entre l'angle obtus (Go) et les deux autres une corrélation négative existe, entre les derniers mutuellement une corrélation positive. Des côtés, la ligne condylomentale (Cd-Mt) est le moins corrélatif avec les angles du triangle mandibulaire, et des angles, Go est le plus corrélatif avec les côtés du triangle.

Entre les côtés d'un triangle et les angles de l'autre de même qu'entre les angles des deux triangles, il n'y a pas de corrélation — avec quelques exceptions voir tableau 18. D'autre part, les côtés d'un triangle présentent de la corrélation avec les côtés de l'autre; pourtant, une corrélation significative dans les deux matériaux craniens n'est obtenue qu'entre la longueur de la base cranienne (Na-Ba) et la ligne condylomentale (Cd-Mt).

Dans le triangle facial la longueur de la base cranienne est le côté le moins variable, l'hauteur supérieure de la face (Na-Pr) le plus variable. Les angles du triangle sont à peu près également variables.

Dans le triangle mandibulaire la ligne condylomentale est le côté le moins variable, l'hauteur de la branche mâchoire (Cd-Go) le plus variable. Des angles du triangle, Go est l'angle le plus variable, Mt le moins variable.

Pour la longueur de la base cranienne (Na-Ba) et la ligne condylomentale (Cd-Mt) des relations intratriangulaires analogues existent. Ainsi, les deux côtés sont assez indépendants de la forme des triangles, c'est-à-dire de leurs angles. De plus, les deux côtés dans le triangle facial et le triangle mandibulaire sont les côtés les moins variables. Finalement, ces côtés sont les seuls qui dans les deux matériaux craniens présentent une corrélation inter-triangulaire significative.

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Table 2.
Co-variations between the sides of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na-Pr			Ba-Pr			Na-Ba		
	DM _{II} -MI	$\frac{DM_{II}-MI}{M_I}$	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI} \cdot 100$	DM _{II} -MI	$\frac{DM_{II}-MI}{M_I}$	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI} \cdot 100$	DM _{II} -MI	$\frac{DM_{II}-MI}{M_I}$	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI} \cdot 100$
Oslo	+ 6.70	+ 9.67	16.75	- 0.92	- 0.94	1.01	+ 0.88	+ 0.86	1.14
Lapp	+ 6.63	+ 10.27	15.42	+ 2.95	+ 3.09	3.35	+ 1.88	+ 1.92	2.99
Oslo	- 0.91	- 1.24	1.23	+ 8.12	+ 8.67	16.57	+ 3.68	+ 3.67	5.58
Lapp	+ 2.50	+ 3.76	3.91	+ 8.44	+ 9.06	15.63	+ 4.42	+ 4.58	8.50
Oslo	+ 0.96	+ 1.33	1.35	+ 4.86	+ 5.11	6.57	+ 6.99	+ 7.06	15.54
Lapp	+ 1.89	+ 2.83	2.82	+ 6.04	+ 6.45	7.84	+ 6.19	+ 6.48	17.20

Table 3.
Co-variations between the angles of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na		Pr		Ba	
	DM _{II} -MI	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI}$	DM _{II} -MI	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI}$	DM _{II} -MI	$\frac{DM_{II}-MI}{\epsilon DM_{II}-MI}$
Oslo	+ 5.87	13.97	- 1.71	3.17	- 3.74	6.80
Lapp	+ 5.47	15.63	- 3.56	7.12	- 1.82	4.33
Oslo	- 2.26	3.71	+ 4.78	14.06	- 2.18	3.57
Lapp	- 4.12	8.58	+ 5.19	14.03	- 0.65	1.48
Oslo	- 3.75	6.94	- 1.74	3.41	+ 5.31	11.07
Lapp	- 2.64	4.80	- 1.30	2.28	+ 4.22	16.88

Table 4.
Co-variations between the sides and the angles of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na		Pr		Ba	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Na-Pr	-2.23 -0.45	3.60 0.76	-1.48 -2.58	2.85 4.23	+3.68 +2.96	8.36 8.22
Ba-Pr	+4.17 +3.56	7.45 7.12	-0.70 -2.67	1.32 4.94	-3.58 -1.08	7.46 2.51
Na-Ba	-0.16 +0.66	0.24 1.12	+2.27 +	4.13 1.71	-2.44 -1.60	4.88 3.81

Table 5.
Co-variations between the angles and the sides of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na-Pr		Ba-Pr		Na-Ba	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Na	-2.98 -0.82	4.02 -1.20	+5.83 +5.80	7.88 7.34	+0.49 +0.47	0.59 0.69
Pr	-2.37 -2.64	3.19 -3.83	-0.70 -4.75	0.80 5.52	+3.13 +0.74	4.23 1.16
Ba	+5.29 +4.96	7.57 7.63	-4.90 -3.03	6.45 3.33	-2.92 -2.11	3.95 3.35

Table 6.
Co-variations between the sides of the mandibular triangle. (Compare with the chapter on "statistical methods".)

Independent variable	Cd-Gc				Go-Mt				Cd-Mt			
	D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}} \cdot 100$		D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}} \cdot 100$		D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}} \cdot 100$	
	+ 7.54	+ 9.17	+ 12.14	+ 17.22	+ 1.48	+ 1.94	+ 2.11	+ 2.65	+ 4.82	+ 5.90	+ 4.03	+ 5.48
Oslo Lapp	+ 7.54	+ 9.17	+ 12.14	+ 17.22	+ 1.48	+ 1.94	+ 2.11	+ 2.65	+ 4.82	+ 5.90	+ 4.03	+ 5.48
Oslo Lapp	+ 2.58	+ 2.91	+ 4.02	+ 5.15	+ 6.94	+ 8.14	+ 9.48	+ 11.74	+ 5.81	+ 7.33	+ 4.06	+ 7.26
Oslo Lapp	+ 4.91	+ 6.77	+ 7.78	+ 12.33	+ 3.80	+ 4.84	+ 5.05	+ 6.03	+ 8.73	+ 11.04	+ 7.43	+ 15.31
												+ 16.17

Table 7.
Co-variations between the angles of the mandibular triangle. (Compare with the chapter on "statistical methods".)

Independent variable	Go				Cd				Mt			
	D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$		D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$		D _{MII-MI}		$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	
	+ 9.79	+ 12.04	15.79	17.45	- 5.33	- 7.04	11.59	14.98	- 4.47	- 6.16	11.18	14.00
Oslo Lapp	+ 9.79	+ 12.04	15.79	17.45	- 5.33	- 7.04	11.59	14.98	- 4.47	- 6.16	11.18	14.00
Oslo Lapp	- 8.50	- 11.60	11.19	14.50	+ 5.96	+ 7.38	14.90	17.17	+ 2.99	+ 5.44	4.46	10.88
Oslo Lapp	- 7.81	- 9.44	9.89	9.63	+ 3.93	+ 4.91	7.02	7.80	+ 5.28	+ 7.49	17.03	18.73

Table 8.

Co-variations between the sides and the angles of the mandibular triangle. (Compare with the chapter on "statistical methods".)

	Go		Cd		Mt	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Cd-Go	-4.09 -4.73	4.30 4.11	+0.45 +1.15	0.70 1.51	+3.48 +4.17	6.96 7.19
Go-Mt	-3.23 -2.81	3.30 2.30	+2.73 +2.39	4.63 3.23	+0.64 +0.54	1.12 0.79
Cd-Mt	+1.97 +1.23	1.88 0.95	-1.59 -0.55	2.48 0.73	-0.32 +0.07	0.58 0.10

Table 9.

Co-variations between the angles and the sides of the mandibular triangle. (Compare with the chapter on "statistical methods".)

	Cd-Go			Go-Mt			Cd-Mt		
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	$\frac{D_{MII-MI}}{M_I} \cdot 100$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	$\frac{D_{MII-MI}}{M_I} \cdot 100$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	$\frac{D_{MII-MI}}{M_I} \cdot 100$
Go	-2.64 -3.48	3.47 3.82	-3.95 -5.82	-2.10 -1.78	2.96 2.28	-2.69 -2.38	+2.57 +2.66	2.82 2.13	+2.14 +2.31
Cd	+0.01 +2.00	0.01 2.08	+0.02 +3.51	+4.20 +2.39	6.56 2.99	+5.58 +3.29	-1.96 -2.43	2.13 2.06	-1.60 -2.06
Mt	+4.43 +5.42	6.07 6.38	+7.02 +9.84	-0.09 +0.75	0.13 0.90	-0.01 +1.02	+0.62 -0.55	0.67 0.47	+0.51 -0.47

Table 12.

Co-variations between the angles of the facial triangle and the angles of the mandibular triangle. (Compare with the chapter on "statistical methods".)

	Go		Cd		Mt	
	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Na	+ 0.56 - 0.45	0.49 0.31	- 0.39 + 0.33	1.39 1.02	- 1.08 - 0.53	1.66 0.63
Pr	- 2.83 - 0.44	2.57 0.32	+ 1.22 + 0.87	1.77 0.95	+ 1.36 - 0.62	2.27 0.76
Ba	+ 0.59 + 2.20	0.55 1.57	- 0.09 - 2.34	0.14 2.89	+ 0.28 - 0.53	0.45 0.67
Independent variable						

Table 13.

Co-variations between the angles of the mandibular triangle and the angles of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na		Pr		Ba	
	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D_{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Go	- 0.24 + 0.24	0.35 0.40	- 1.74 - 0.88	3.05 1.44	+ 1.15 + 0.77	2.13 1.64
Cd	+ 0.16 - 0.02	0.23 0.03	+ 1.70 + 0.43	3.04 0.68	- 0.97 - 0.53	1.76 1.06
Mt	- 0.24 + 0.22	0.33 0.36	+ 0.65 - 0.20	1.12 0.31	+ 0.33 - 0.15	0.57 0.33
Independent variable						

Table 14.

Co-variations between the sides of the facial triangle and the angles of the mandibular triangle. (Compare with the chapter on "statistical methods".)

	Go		Cd		Mt	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Na-Pr	+ 0.31 + 1.70	0.29 1.30	+ 0.35 - 1.81	0.50 2.23	+ 0.64 - 0.75	1.07 1.01
Ba-Pr	- 0.81 - 0.54	0.74 0.40	+ 0.32 + 0.31	0.48 0.37	+ 0.86 + 0.25	1.37 0.32
Na-Ba	- 1.78 - 0.62	1.82 0.47	+ 1.23 + 0.33	1.84 0.41	+ 1.28 + 0.47	2.29 0.64
Independent variable						

Table 15.

Co-variations between the angles of the mandibular triangle and the sides of the facial triangle. (Compare with the chapter on "statistical methods".)

	Na-Pr		Ba-Pr		Na-Ba	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Go	- 0.03 + 0.51	- 0.04 0.70	- 1.58 - 0.03	1.78 0.03	- 2.59 - 0.80	3.33 1.23
Cd	- 0.40 0.00	0.53 0.00	+ 0.49 + 0.20	0.53 0.20	+ 1.27 + 0.60	1.59 0.88
Mt	+ 1.26 + 0.26	1.66 0.33	+ 0.01 + 1.83	0.01 1.93	+ 0.72 + 1.35	0.87 1.96
Independent variable						

Table 16. *Co-variations between the sides of the mandibular triangle and the angles of the facial triangle. (Compare with the chapter on "statistical methods".)*

	Na		Pr		Ba	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Cd-Go	+ 0.05	0.08	+ 0.49	0.83	- 0.37	0.69
	- 0.18	0.31	- 0.55	0.98	+ 0.74	1.57
Go-Mt	- 0.28	0.41	+ 0.67	1.18	- 0.67	1.16
	0.00	0.00	- 0.75	1.29	+ 0.75	1.60
Cd-Mt	+ 0.78	1.10	- 0.81	1.42	- 0.46	0.81
	- 0.88	1.38	- 0.65	1.02	+ 1.47	3.06

Table 17. *Co-variations between the angles of the facial triangle and the sides of the mandibular triangle. (Compare with the chapter on "statistical methods".)*

	Cd-Go		Go-Mt		Cd-Mt	
	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$	D _{MII-MI}	$\frac{D_{MII-MI}}{\epsilon D_{MII-MI}}$
Na	+ 0.38	0.43	+ 0.07	0.09	+ 0.84	0.85
	- 1.17	0.74	+ 1.10	1.16	- 2.62	1.91
Pr	+ 0.93	1.08	+ 0.60	0.80	- 1.63	1.66
	- 1.27	1.16	- 2.51	2.85	- 3.30	2.62
Ba	- 0.78	0.91	- 1.13	1.46	- 1.27	1.38
	+ 2.82	2.56	+ 0.16	0.22	+ 4.71	6.54

Table 18.

Intra-racial relations between the different components of the facial and mandibular triangles, obtained by summation of the results of the analysis in the Oslo and Lapp materials.

	Na-Pr	Ba-Pr	Na-Ba	Na	Pr	Ba	Cd-Go	Go-Mt	Cd-Mt	Cd	Go	Mt
Na-Pr		(+) ?	(+) ?	(-) ?	(-) ?	+	(+) ?	(+) ?	(+) ?	0	0	0
Ba-Pr	(+) ?		+	+	(-) ?	-	0	0	(+) ?	0	0	0
Na-Ba	(+) ?	+		0	(+) ?	-	(+) ?	(+) ?	+	0	0	0
Na	(-) ?	+	0		-	-	0	0	0	0	0	0
Pr	(-) ?	(-) ?	(+) ?	-		(-) ?	0	0	0	0	(-) ?	0
Ba	+	-	-	-	(-) ?		0	0	(+) ?	0	0	0
Cd-Go	(+) ?	0	(+) ?	0	0	0		(+) ?	+	0	-	+
Go-Mt	(+) ?	0	(+) ?	0	0	0	(+) ?		+	+	(-) ?	0
Cd-Mt	(+) ?	(+) ?	+	0	0	(+) ?	+	+		(-) ?	(+) ?	0
Cd	0	0	0	0	0	0	0	+	(-) ?		-	+
Go	0	0	0	0	(-) ?	0	-	(-) ?	(+) ?	-		-
Mt	0	0	0	0	0	0	+	0	0	+	-	

- + : significant positive correlation
- : » negative »
- (+) : probable positive »
- (-) : » negative »
- (+)?: possible positive »
- (-)?: » negative »
- 0 : no correlation

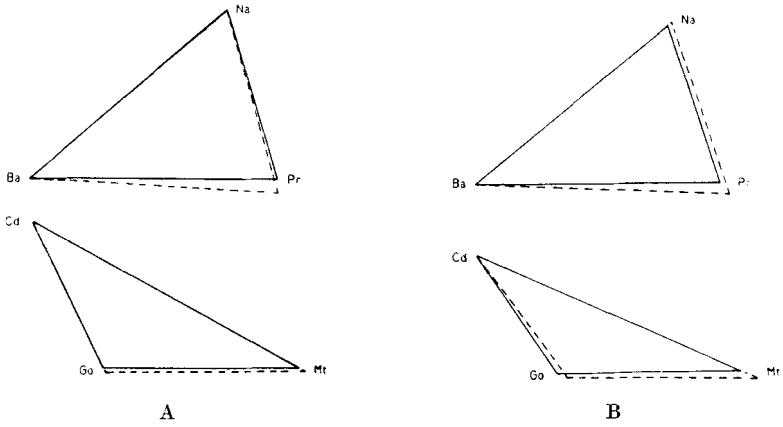


Fig. 3. Changes in the profile patterns at the same time as the independent variable Na-Pr. A. Oslo material. B. Lapp material.

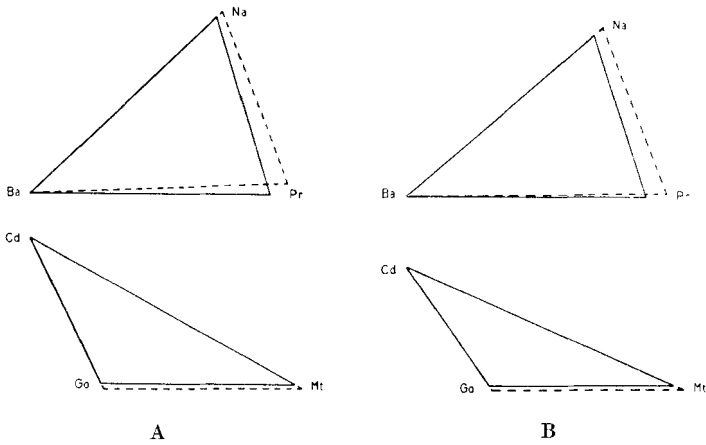


Fig. 4. Changes in the profile patterns at the same time as the independent variable Ba-Pr. A. Oslo material. B. Lapp material.

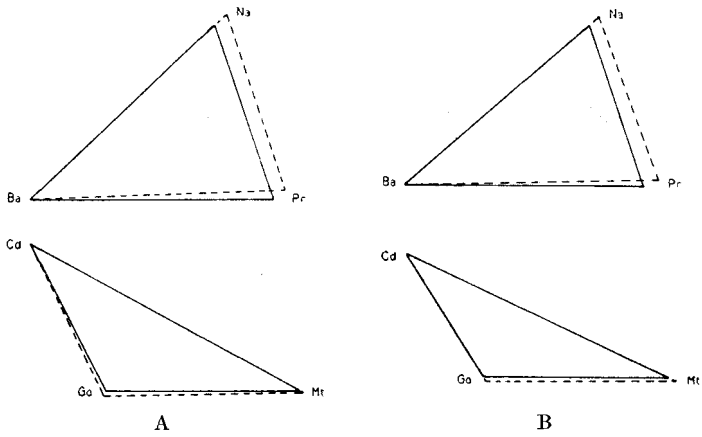


Fig. 5. Changes in the profile patterns at the same time as the independent variable Na-Ba. A. Oslo material. B. Lapp material.

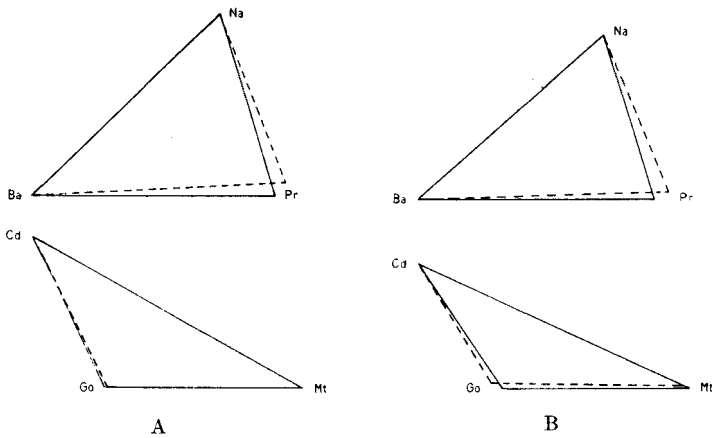


Fig. 6. Changes in the profile patterns at the same time as the independent variable Na. A. Oslo material. B. Lapp material.

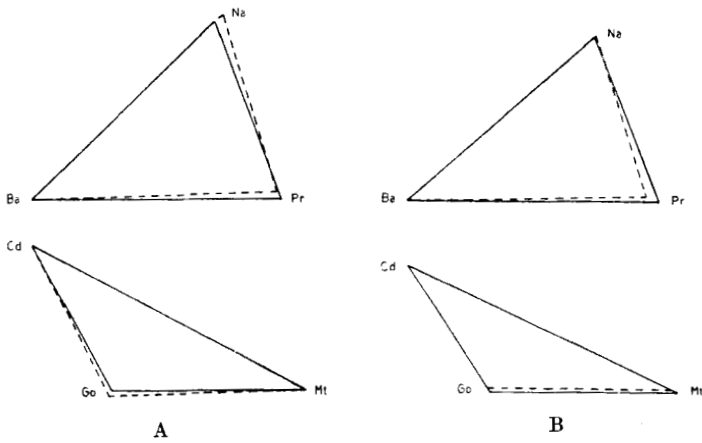


Fig. 7. Changes in the profile patterns at the same time as the independent variable Pr. A. Oslo material. B. Lapp material.

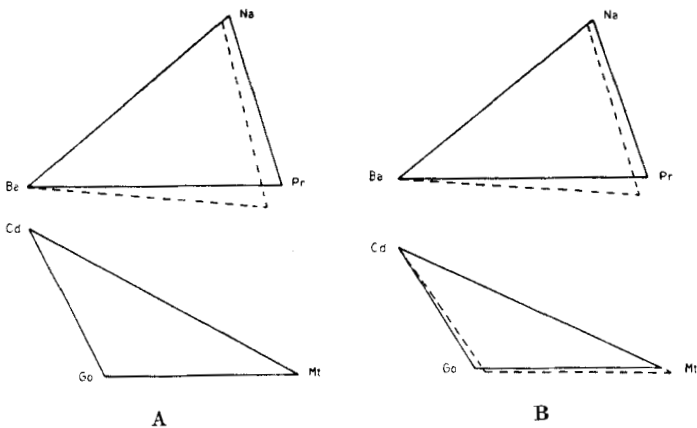


Fig. 8. Changes in the profile patterns at the same time as the independent variable Ba. A. Oslo material. B. Lapp material.

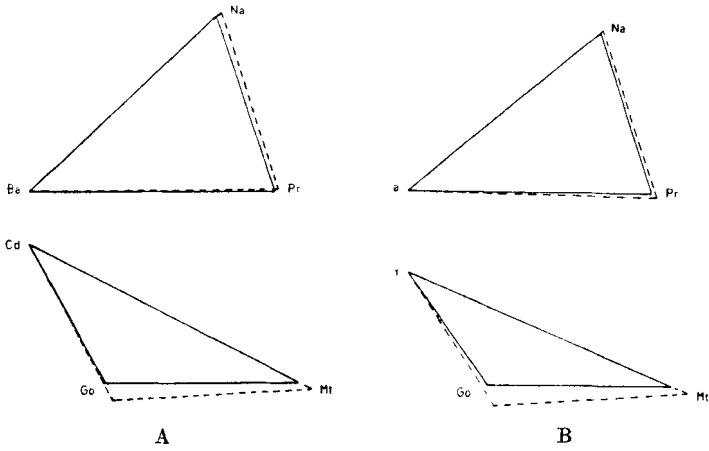


Fig. 9. Changes in the profile patterns at the same time as the independent variable Cd-Go. A. Oslo material. B. Lapp material.

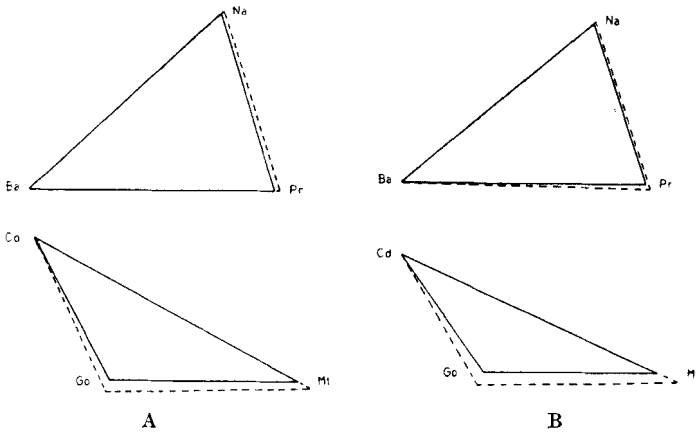


Fig. 10. Changes in the profile patterns at the same time as the independent variable Go-Mt. A. Oslo material. B. Lapp material.

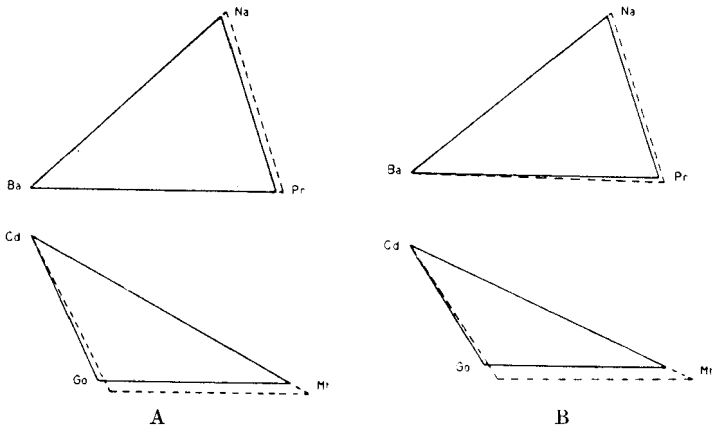


Fig. 11. Changes in the profile patterns at the same time as the independent variable Cd-Mt. A. Oslo material. B. Lapp material.

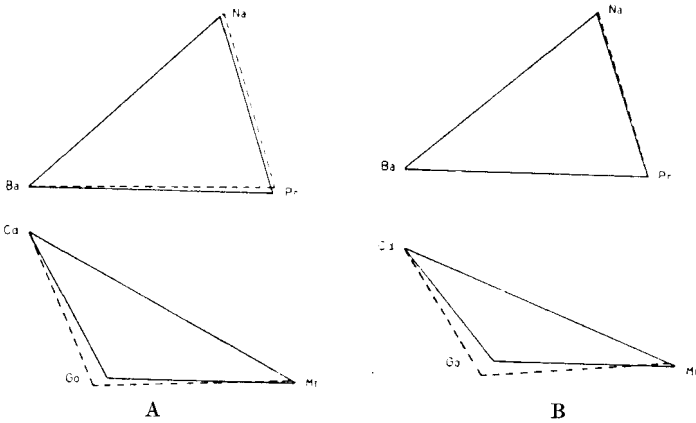


Fig. 12. Changes in the profile patterns at the same time as the independent variable Cd. A. Oslo material. B. Lapp material.

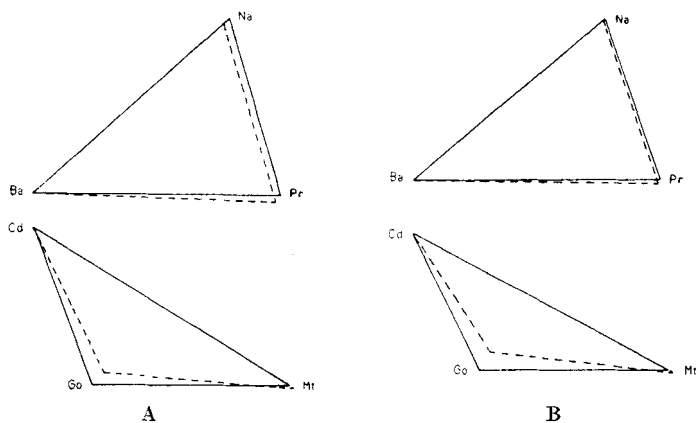


Fig. 13. Changes in the profile patterns at the same time as the independent variable Go. A. Oslo material. B. Lapp material.

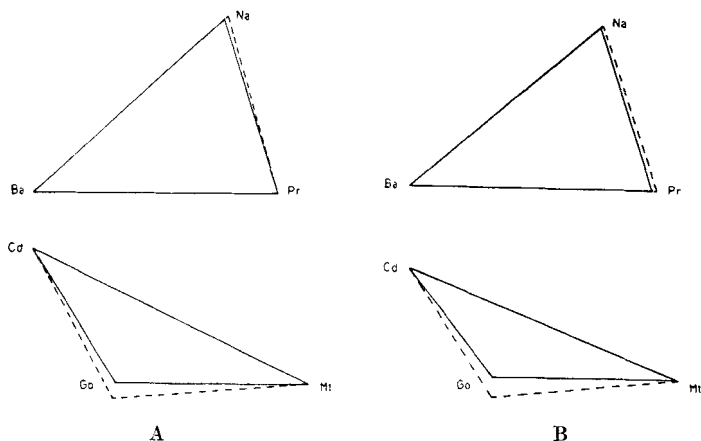


Fig. 14. Changes in the profile patterns at the same time as the independent variable Mt. A. Oslo material. B. Lapp material.

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