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THREE-DIMENSIONAL INTRA-ORAL RADIOGRAPHY

by

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General aspects

The structure of the radiograph is dependent on a number of factors falling under two main headings: physical and geometric.

The physical factors comprise the spectral distribution of the ray, its energy, the properties of the film or plate, and the developing process. The geometric factors are determined by the position (shape, dimensions and structure) of the object, film, and other components in relation to the X-ray source.

A thorough study of the structure of the radiograph, however, must take into consideration not only the physical and geometric factors but also the subjective factors associated with the viewer, and the correct use of the radiograph requires an intimate understanding of the influences of these various factors.

A radiograph is the result of a unique set of conditions of projection, in which the ray-source, the object and the film bear a particular relationship to one another in space. A definition of this relationship must be expressed in geometrical terms. The simplest way to define this particular relationship is to consider the focus as a point, and the radiographic image as composed of points that correspond to rectilinear ray paths from the focus. Simple though this conception is, it is wide enough to cover the essentials of practical diagnostics. If, however, a more theoretical insight is required as the basis for interpreting the radiograph, this simple approach will be inadequate. It is evident, for instance, that the focus is not really a mathematical point, and that the radiographic image cannot be composed of points.

The geometry of the radiograph, when the ray-source is conceived as a point, may be studied from the aspect of perspectography, and an analogy may be drawn — albeit cautiously — with photography. It is true that in photography a lens with certain optical properties is used, but this is of no real significance since the analogy is restricted to the geometric properties.

To understand what has been written on the geometry of radiographic analysis in relation to dental diagnosis, it is necessary to bear in mind the development of odontology and dental equipment. The procedure for radiographic examination of the teeth and jaws has not been developed directly from theoretical principles. On the contrary, clinical needs have given rise to disconnected geometric rules for the application of radiography in dental diagnosis. For a long time the only metric property required of a radiograph was that the image of the tooth should be as long as the tooth itself; with practice this could be obtained approximately by means of the bisector or isometric rule. More recently the demands on the metric properties of the radiographic image have become stricter.

In 1930 *Herulf* and *Wallgren*, in an unpublished paper, endeavoured to widen and introduce some coherence into the theory of projection applied to the teeth. They deduced general formulae for the position of the focus for certain specified metric properties of the radiograph in the orthoradial plane. In his postgraduate courses in radiographic diagnostics *Herulf* demonstrated a more advanced technique theoretically as well as experimentally.

Later (1952) *Herulf* evolved a general formula for the outer orientation in radiography, in accordance with the bisector rule.

Another approach to radiographic projection is based on projective geometry. This makes it possible to obtain a systematic and uniform conception of the three-dimensional properties of radiographic projection. An attempt is made here to provide a geometric analysis of this form of projection.

The basic concept is central projection, which means that straight lines from a fixed point, the centre of projection, are imagined as passing through the object (at object points) and intersecting the image plane at points (the image points). The application to photography, and especially to radiography, is simple to follow; in the latter, the object point must lie between the centre of projection (focus) and the image point (Fig. 1). *Projective geometry* deals with projections of this type. It is abstract, in as much as only projective properties are treated,

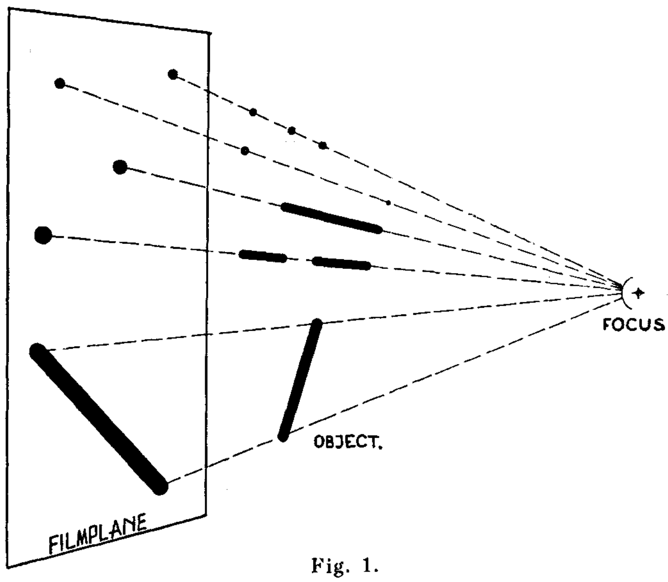

Fig. 1.

the metric properties of the geometric figures being disregarded. It is the geometric properties, however, that are generally of the greatest practical interest. For this reason, at the moment of exposure the centre of projection in relation to the film and the object must be known; shape, size, and other geometric properties can then be studied. This implies a restriction, a special case within otherwise general projective geometry. (With a restricted number of dimensions of the object the limitation becomes less strict.) Thus, the form of the central projection — i.e. the relative position between the image plane and the centre of projection — must be known. Each projected ray path passes through the centre of projection, and through its image point. It follows that the position of the centre of projection and the image points together determine the form of the projection. This relative position is generally called the *inner orientation,* to distinguish it from the *outer orientation* (i.e. the position of the central projection as a whole in relation to the object). By applying limiting conditions, simpler forms of projection can be obtained, for instance the reproduction of an area in the object plane as a similar area in the parallel image plane. Moreover, approximately parallel projection (affine reproduction) is obtained if the object subtends a very small angle at the focus. Affine reproduction has the special property of proportionality between uni-directional lengths, and parallelism is retained. Angles and distances are, on the other hand, not necessarily unchanged.

Image perspective

If a radiograph is viewed from the position of the projection centre at the time of exposure, a correct impression of the perspective will be obtained; any other position of the eye will give a false perspective. If the position of the eye is known, so are the type and degree of the error in the perspective. Radiographs may be employed for *viewing* only, or for *viewing and measurement.* In the former case it is preferable to place the eye at the centre of projection, or, at least, where the perspective does not disturb the reading of the radiograph. For viewing and measurement the subjective impression is of no importance, since readings obtained by measurement on the radiograph may be corrected for errors in perspective if the position of the projection centre is known. In both cases the inner orientation must therefore be known. Disregard of this condition is probably due to the fact that in photography the projection centre is generally placed directly in front of the image (on a perpendicular from the centre of the photograph) and that quite large errors in perspective are tolerated, at least so long as no attempt is made to estimate the proportions of the reproduced object with the eye. There may be a considerable error in perspective in the radiograph taken by the free technique. This is one of the disadvantages of this method. The risk may be as great in any other method if the sources of error are disregarded.

In photography the inner orientation of the camera is fixed; the reproductive properties of the lens are known, as well as its position in relation to the image, which is usually a plane. In view of what has been said about the inner orientation, the shape of the film must be known. The plane form is the most practical and the easiest to reproduce. Many cameras have a variable range adjustment and thus a different inner orientation for any given distance. The advantage of determining the inner orientation once and for all is obvious — it need not then be calculated for each exposure made. There are corresponding advantages in radiography. The apparatus must then be constructed like the camera, the inner orientation being either fixed or adjustable. This requires a connection between film and focus. This connection must be fixed or adjustable yet permitting measurement. As with the camera, and for the same reason, it is most convenient to keep the film plane, and in any case not to change its shape. This is effected by using a film holder.

There are various opinions as to the side of the radiograph the image should be viewed on. In transmitted light often the only possibility of viewing the image is from the side opposite the focus, and many prefer this method for studying intra-oral radiographs. For accurate interpretation of the radiograph due regard must be paid to the resulting change in perspective.

The importance of stability

When photographing moving objects with too long an exposure a blurred image results. In intra-oral radiography, when the free technique and the usual apparatus are employed, the risk of such indistinctness is considerable, since an exposure of some seconds is not uncommon and both patient and X-ray machine are movable. This disadvantage may be eliminated by having patient, film and X-ray machine in a fixed relative position. This may be effected simply by means of a piece of thermoplastic compound which the patient bites on and which then hardens in the mouth to give an impression of the teeth. The compound is attached to a film holder which is fixed to the X-ray apparatus. A carefully made impression normally permits repeated exposures with identical geometric relationship. (A difficulty may arise if the dentition is abnormal, with many or all teeth missing, for example, or if there are such long intervals between exposures that the dentition may have changed in the meantime.)

The fixed relative position of the centre of projection, the object and the film, is the essence of the procedure known as periodic identical radiography. This relationship is of value in stereo-radiography. Here the inner orientation is altered between the two exposures. When viewed stereoscopically, such stereopairs give a three-dimensional effect, which is of obvious value in their interpretation. The details of the radiographs can then be better arranged for perception and are more easily distinguished, and moreover it is often important to obtain a general impression of the arrangement of the details in space. Although it is often possible to make satisfactory diagnosis by comparing several separate radiographs, the same deductions may be made automatically and with greater reliability from the three-dimensional model. In addition to this, more may be read from the radiographs by reason of the greater possibility of identifying details from image to image in stereoscopic viewing. As will be shown below, stereopairs may be used for locating all objects visible in the two component radiographs, which means that all questions of shape, size, angle, etc. may be resolved.

The central ray

The X-ray machine component for adjusting the beam in the free technique consists of a bakelite cone, the axis of which is generally called the central ray, because it passes through the centre of the primary diaphragm. The shape of the cone and the term "central ray" suggests that the image might exhibit symmetry of rotation about the central ray. The conception of the central ray is primarily of value for positioning the focus in relation to the film and the object — i.e. for roughly determining the inner and outer orientations. The diaphragm is then adjusted

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Instructions for viewing the plates on the following pages.

People with normal sight can learn to view a double image stereoscopically without the aid of a stereoscope. The eyes must be accommodated for a near point without the optic axes being convergent. This is easily accomplished by looking at the images with the eyes on each side of a sheet of paper, the paper being held so that each eye is able to view only one of the two pictures.

Illustrations of stereo-radiographs

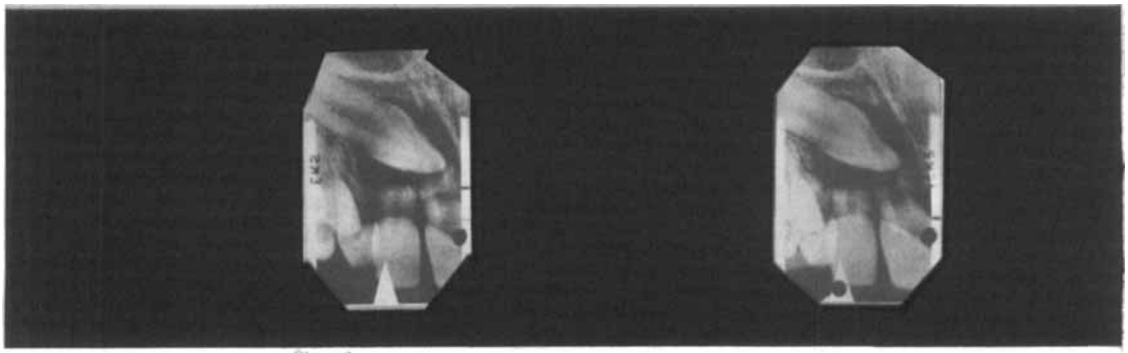
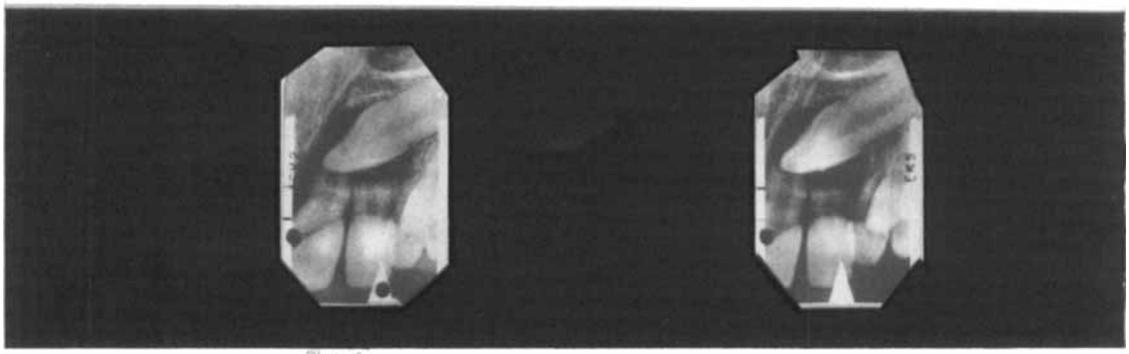


Plate 2



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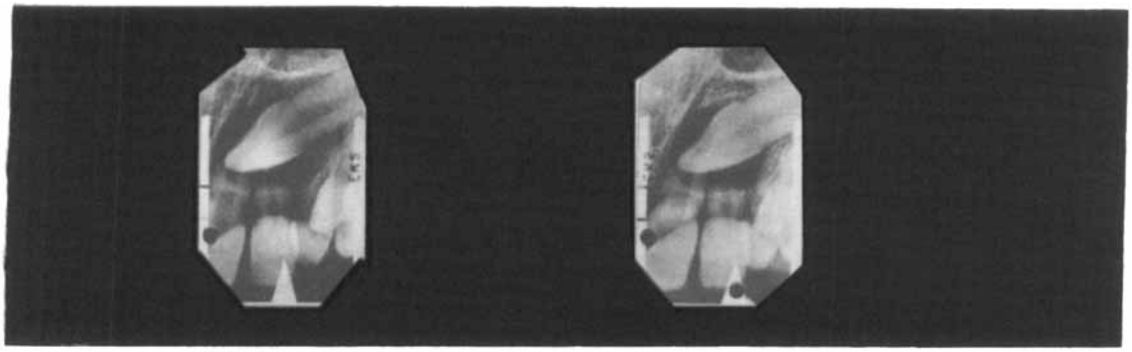

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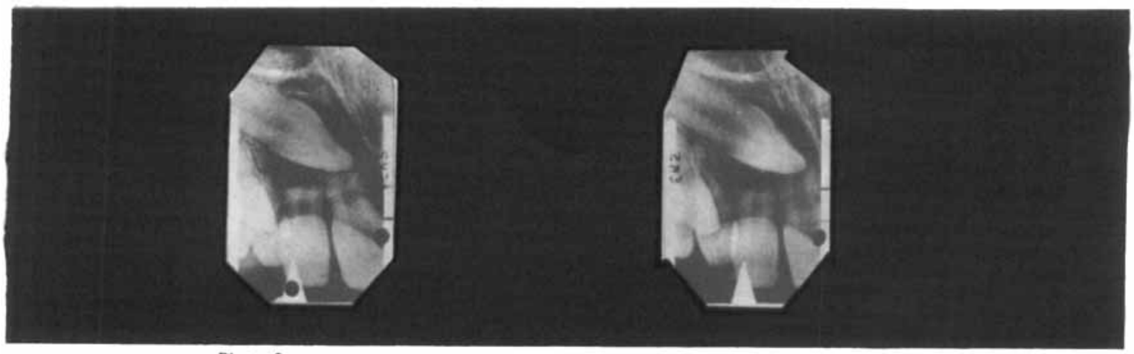

Plate 5

in relation to the central ray so that the beam is incident on the object and film. *On exposure the central ray may be chosen anywhere within the solid angle determined by the diaphragm.*

Intra-oral radiography by the free technique is generally performed according to a rule of isometry, i.e. the bisector rule, by which the orientation is fixed so that linear distances — the length of the tooth, for instance — are reproduced without scale error. The distance to be correctly represented on the film may be from one point in the film plane (the incisal edge) to another point (the root tip) that cannot be brought simultaneously into the plane of the film. For the reproduction of this distance to the scale of 1: 1 — that is, in conformity with the bisector rule — the ray from the tube focus that images the root tip must be perpendicular to the bisector of the angle between the line concerned (the tooth axis) and the film plane.

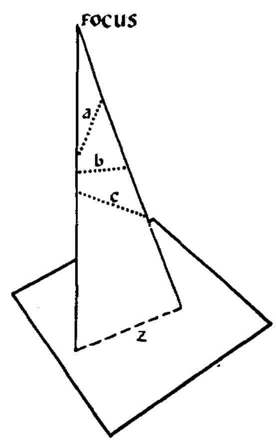

Fig. 2.

It should be observed that no other distance *on the same line* is reproduced to the scale of 1: 1, as there is only one ray perpendicular to the bisector. Since, however, any point of the image may be considered as the sum of the images of the infinite number of object points on the straight line connecting the focus and the image point concerned, there is an infinite number of distances *in space* that can be reproduced to the scale of 1: 1 (Fig. 2).

Let us imagine the ray path through the object as consisting of an infinite number of object points. These will be of various degrees of radiolucency. The image point due to this particular ray will be the result of the integrated effect of all these object points. The more radiopaque object points, however, will "overshadow" the others and thus dominate the film. To the viewer the image point will then represent the dominating object point.

Now let us consider two such rays producing two separate image points; these image points may be due to two "dominating" object points, one on each ray. It will be mere chance if these object points are the same distance apart as the image points on the film. This means, then, that it is pure coincidence if a certain distance in the object is reproduced to the scale 1: 1 in the radiograph. To ascertain whether this distance is correctly reproduced, a knowledge of the relative position of the points in space is necessary, and this cannot be obtained from the radiograph. The isometric (bisector) rule requires this to be known in advance (on exposure). If the positions of the extreme points of the length cannot be seen or otherwise established at the time of exposure, the necessary data cannot be obtained. The object of the exposure is generally not to determine the length of the tooth; it is rather that the correct length of its image should guarantee the correctness of the image in other respects. For reasons already stated it is impossible to draw any conclusions from this by means of the isometric rule alone; the question is still one of perspective. If the radiograph is required for measuring object distance, a knowledge of the properties of the perspective of the image is essential.

To sum up: it is generally difficult to use the isometric rule (bisector rule) for a specific object distance (a tooth, for instance) since the root tip is invisible. It cannot be seen from the radiograph how far, at the time of exposure, the requirements of the isometric rule were satisfied for the selected object distance. Finally, the value of the radiograph for determining distances in the object cannot be judged with the aid of the isometric rule alone, as its application is dependent on a more complete knowledge of the nature of the perspective of the image.

Scale determination

In X-ray work a reference body of heavy material and known length, e.g. a rod, is sometimes radiographed with the object so as to define the scale of the radiographic image. It should be remembered here that the terms "scale", "degree of magnification", etc., have a meaning only in the case of figures that are similar; the ratio between two lengths may only be considered as a scale factor when they are corresponding lengths in similar figures.

Since the reproductions of these figures are similar, a scale factor can be calculated that is applicable to all figures in the plane containing the reference body and parallel to the film, if the reference body is placed parallel to the film. The value of this procedure is limited by the fact that the single radiograph does not indicate which image points correspond to object points in this plane; this must be ascertained separately. If, on the other hand, the reference body is not placed parallel to the film, the significance of the term "scale" is lost, since the image is formed according to the laws of perspective.

"Long distance radiography" is a term applied to a type of radiography where large distances from focus to object and image plane are used, partly to gain sharpness and partly to simplify the geometry of the reproduction. Since the first of these objectives entirely depends on the angle that the focus subtends at the object, and the second entirely depends on the angle that the object subtends at the focus (the optimal position of the image in relation to the object is chosen in advance), it is clear that an intra-oral radiograph with a small target area can very well be geometrically equivalent to a long distance radiograph of, for example, a whole head.

Insertion of a co-ordinate system)*

The distance between two points is most simply determined by placing a ruler along the line joining the points. This is not always possible in practice. Here the procedure becomes more involved, several measurements being required from which the distance is obtained by calculation. A simple example of this may be of value.

Let us suppose that it is possible to measure in a plane containing the two points A and B. A visual picture can then be easily obtained on a sheet of paper placed in the plane (Fig. 3).

If, on the other hand, it is only possible to place the linear measure in two mutually perpendicular directions in the plane, the determination of AB is still feasible. The case is illustrated in Fig. 4. The one permissible direction is parallel to AC, and

Paragraphs indicated with *) may be omitted by readers with some knowledge of mathematical physics.

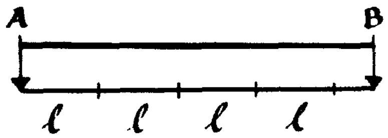

Fig. 3.

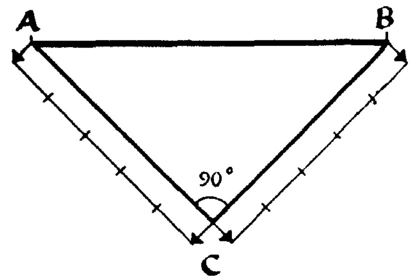

Fig. 4.

the other parallel to CB, these being mutually perpendicular. AC and CB are measured as before, and AB determined from these by calculation.

Distances in one plane can be determined in a perpendicular co-ordinate system where each point is defined by its distance from two perpendicular axes. (These distances are called co-ordinates.)

By expressing their co-ordinates in one co-ordinate system it is possible to define systematically and clearly the positions and configurations of a large number of points. Although their mutual positions can be defined by the geometric shape formed by the points, this would in many cases be very complicated, especially when there are many points and their distribution is irregular.

Placed in the co-ordinate system, the figure appears as shown in Fig. 5.

The right-angle co-ordinates for A are the distances Oa_1 and Oa_2, and the co-ordinates for B the distances Ob_1 and Ob_2. Since $AC = a_1b_1$ and $CB = a_2b_2$,

$$(AB)^2 = (a_1b_1)^2 + (a_2b_2)^2 \qquad (1$$

Suppose the co-ordinate system is placed as shown in Fig. 6.

It is clear that the right-angled triangle ABC^1 at C^1 has changed in shape. The co-ordinates of the points A and B are now Oa^1_1 and Oa^1_2, Ob^1_1 and Ob^1_2 are completely new values.

$$\text{Then } (AB)^2 = (a^1_1b^1_1)^2 + (a^1_2b^1_2)^2. \qquad (2)$$

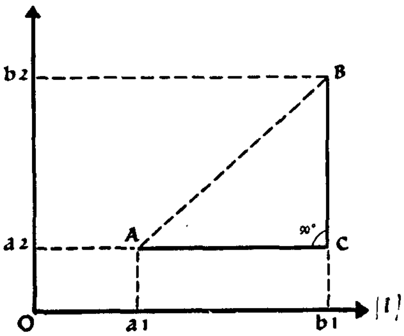

Fig. 5.

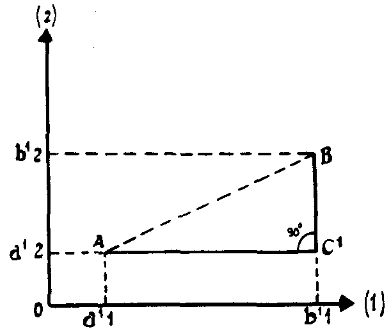

Fig. 6.

A comparison of the expressions (1) and (2) shows that the distance AB is independent of the position of the co-ordinate system. The procedure for calculation is always the same, i.e. the solution of AB from an equation of the above type (1).

Further rotation of the co-ordinate system until its axis (1) is parallel to AB gives a^1_2 and b^1_2 coincident; viz. the distance $a^1_2b^1_2$ is zero, and (2) becomes

$$(AB)^2 = (a^1_1b^1_1)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3),$$

giving $AB = a^1_1b^1_1$.

By this means, the case with only two permissible directions of measurement is reduced to one where a ruler can be placed directly along the line AB. The interesting point about this is the revelation of a fact of exceptional importance to the mathematical treatment of physical events. A comparison between expression (3) and expressions (1) and (2) shows that the first is considerably more easy to manipulate. This advantage is gained by choosing a system of co-ordinates in which one axis is parallel to the line AB. As has been pointed out above, AB is of fixed length, i.e. it is uninfluenced by changes in the co-ordinate system. Although it is possible to determine AB in any system it is particularly simple in the system chosen, which gives expression (3). In mathematical physics we are frequently concerned with magnitudes that are independent of the co-ordinate system: magnitudes such as *length, area, volume, pressure*, etc. Such invariants can certainly in theory be calculated in an arbitrary system; with a suitable choice of system, however, their calculation can be simplified, often so as to permit the solution of a problem that would otherwise be insoluble.

The argument may be extended to three or more dimensions, in which case the co-ordinate system is enlarged with more axes through O perpendicular to the others.

Application to radiography

The treatment of the geometry of radiography can be carried out in any system, but in the simple, "normal case" — coincident image planes equidistant from each of the two positions of the focus for the stereo-pair — the choice of a particular system will simplify the evolution of expressions suitable for the practical measurements and calculations.

Evolution of the formula. (Fig 7)

The co-ordinate system consists of three mutually perpendicular axes (1), (2) and (3) with the origin at the focus R_1 and so placed that the axis (2) is perpendicular to the image plane, and the axis (1) passes through the focus R_2. The ray from R_1 that reproduces a certain object point is represented by the two vectors* M and S, which begin in R_1 and end in the object and image points respectively. (The letters M and S are placed in the figure at the end of the respective vectors.) This is expressed vectorially by the formula $M = \mu S$, where μ is a number indicating the ratio of the magnitudes of M and S.

This vector expression corresponds to the three co-ordinate expressions

$$m_1 = \mu s_1$$
$$m_2 = \mu s_2$$
$$m_3 = \mu s_3$$

The value of the ratio (μ) is obtained from the figure when a construction line is drawn from the point S parallel to the ray

* A vector is a quantity that can be represented geometrically by a line having length and direction. Vectors can be used to define the positions of points in space, the vectors being imagined as extending from a fixed origin to all the points.

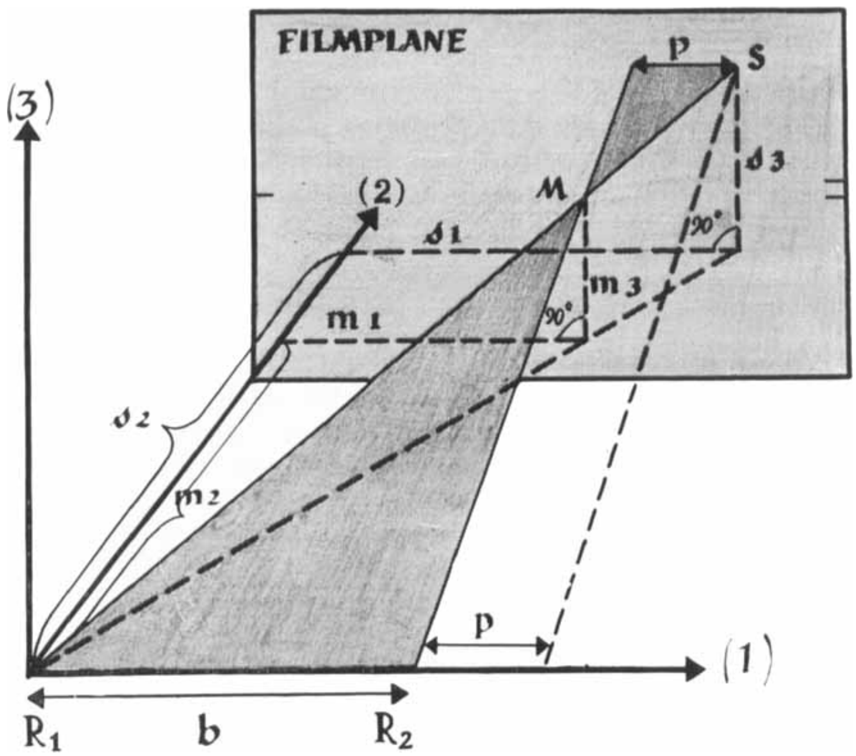

Fig. 7.

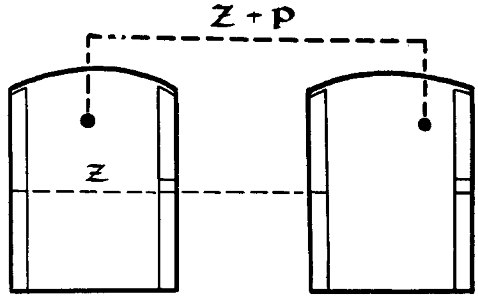

Fig. 8.

from R_2 that reproduces the object point M in the image plane at a distance p from S. We then have $\mu = \frac{b}{b + p}$.

To determine M it is necessary to know b, p, s_1, s_2 and s_3. b and s_2 are constants of the apparatus. p, s_1 and s_3 are measured on the radiograph for a particular object point; for this measurement the inner orientation must be known, so that the co-ordinate system can be placed as indicated.

Practical mensuration

It can be seen from Fig. 7 that the three distances p, s_1 and s_3 to be measured for each point are in the plane of the film. To measure s_1 and s_3 it is necessary to know where s_2 intersects the film; from this point, and parallel to b, the line along which s_1 is to be measured is drawn. This is done by projecting on the film a reference mark of the apparatus — notches on the frame of the film-holder are convenient. From one of these reference marks the intersection of s_2 with the film is found on every exposed radiograph. One way of doing this is by transferring the two marks and the point of intersection on to a piece of paper by tracing (knowing the inner orientation); for each radiograph the marks on the paper are superimposed on the marks on the film, the third point then being transferred to the radiograph. The line joining the marks can be used for adjusting the direction of measurement on the film. (These directions are generally parallel or perpendicular.)

For the measurement of p, let us suppose that the two stereoscopic exposures have been made on the same film. If the object point is sufficiently isolated, its two images are seen on the film. The distance between these image points is p, and it can be measured directly. If the exposure is made instead on different films, and if it is possible to place the radiographs in identical positions in the co-ordinate system (that is, superimpose them so that the reference marks coincide), p can be measured directly with sufficiently strong transmitted light. If such a measurement is difficult to carry out in practice, the same distance can be measured just as well with the stereopairs side by side. If the films are thus placed exactly over each other and from this initial position one of them is moved parallel to the direction of p until the films no longer overlap, all points in the moving image will have moved equal distances — the distance between corresponding frame marks (see Fig. 8).

On measuring both Z (between corresponding frame marks) and Z + p (between corresponding image points) p is obtained by subtraction.

These measurements can be performed without resorting to stereoscopic viewing. The value of the stereoscopic technique in making the measurement is found in the increased precision and the easier identification of corresponding image points, so that more points may be measured. Special photogrammetric instruments will have to be used when a high accuracy of measurement is required.

Polar co-ordinates

A variant of the method involving the perpendicular co-ordinate system is the following, which is applied when only simple measuring appliances are available.

The perpendicular co-ordinate system in Fig. 7, in which s_1 and s_3 are to be measured, can be replaced by a polar co-ordinate system in which a point s is defined by its distance from the origin O and the angle v between Os and a reference line Ou (Fig. 9).

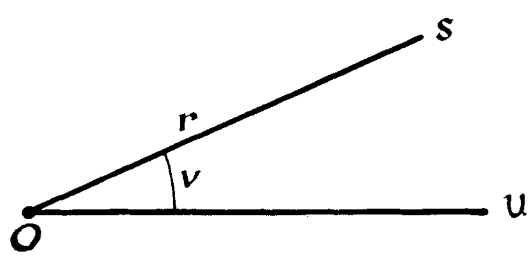

Fig. 9.

From a point O in a plane defined as above, a normal to the plane is drawn, and the length specified along this normal to a point R in space. If one side of the plane is taken as positive and the other as negative, the point will be defined and the system may replace the tri-axial perpendicular co-ordinate system.

If a system of this type is inserted in Fig. 10, with O placed at the point of intersection with s_2 and with u in the parallel to direction of the base (the direction of the axis of the former system (1), it is seen that the normal to the film plane from each reproduced object point (M) must fall on the radius r to the corresponding image point S, (since the plane that contains the focus, object point, image point and point O, is perpendicular to the image plane).

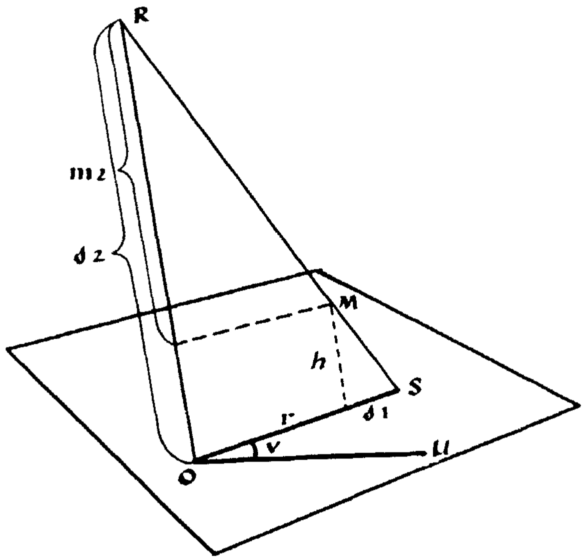

Fig. 10.

To define the object point M it is thus sufficient to determine (a) the point s_1 on r where the normal from M to the image plane meets r, and (b) the length of this normal, h. A simple figure (Fig. 10) shows that

$$h = s_2 - m_2$$

but $m_2 = \mu s_2$,

thus $h = (1 - \mu) s_2 = \frac{p}{b + p} s_2.$

Further, we obtain

$$\frac{s_1 S}{r} = \frac{h}{s_2} = \frac{p}{b+p}$$

or $$s_1 S = \frac{pr}{b+p}$$

and $$h = \frac{ps_2}{b+p}$$

A simple means of defining M is thus provided. P is measured by the procedure described above, and $\frac{p}{b+p}$ is calculated. b, s_2 and the position of O in the film are known (inner orientation), and r is measured on the film. The above formulae give s_1, S and h. This simple yet mathematically acceptable method provides a means of studying directly and graphically the accuracy of reproduction of any point in a radiograph. The method constitutes a suitable starting point for the derivation of approximate methods of mensuration of various kinds.

Determination of the inner orientation

Since the structure of a radiographic image is dependent on the inner orientation, the radiograph itself may be used for determining the inner orientation. The data to be determined are, as shown above, the point O (the intersection of the normal s_2 from the focus to the film) and the length s_2 (the focus-film distance). (In the stereo-pair the length b is required. This can be obtained by comparing the inner orientation for the two component films of the pair.) If two points in space with positions known and fixed in relation to the film are projected, these values can be determined from measurements on the film. If the film holder is provided with these two reference points, the inner orientation can be calculated for each exposure, independently of the position of the focus. Since in stereoradiography it is convenient to use the same focus-film distance for the two component films of a stereo-pair, an arrangement where the focus is fixed in relation to the film is preferable, so that the inner orientation is obtained more easily and the risk of indistinctness due to movement is eliminated. The advantages are so great that it is preferable to use the fixed focus even for single radiographs. Thus, if the focus-film distance (s_2) is standardized, there remains for the inner orientation the position of the point O, which can be obtained from the film if *one* point in space with position fixed in relation to the film is projected with the object.

Practical application

Let us now consider the problem of finding out whether a certain configuration of points in an object has remained unchanged over a period of time. Two principal courses may be followed. Determination is made of the relative distances, angles, areas, volumes or other characteristic properties of the configuration in the two cases to be compared; whether or not these quantities have remained unchanged can be decided by comparing the respective magnitudes in pairs. The other way is to insert a co-ordinate system in the two cases, so that the co-ordinates for all the unchanged points may retain their values while all the changed points take new co-ordinates. The magnitude and the direction of the changes are obtained directly from the differences in the co-ordinates. The second method is preferable if the number of points is large. Otherwise the former method should be used, as the arithmetical work involved in inserting the co-ordinate system is thus avoided.

Accuracy *)

The accuracy of measurement influences the choice of method for mensuration and calculation. In general, the simplest of the sufficiently accurate methods will be chosen for the work. A simple illustration of this is the problem of determining the length of a line. Fig. 5 illustrates a case with two permissible perpendicular directions of measurement, the unknown distance forming a small angle with one of the reference axes. New notation for the co-ordinate differences from which AB is to be calculated is now introduced. AB is denoted by L, a_1b_1 and a_2b_2 by L_2. Formula (1) is then written

$$L^2 = L^2_1 + L^2_2 *,$$

* In this and the following formulae only the inferior figure is an index. The superior is the exponent.

which is solved for L:

$$L = \sqrt{L^2_1 + L^2_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since L_2 is small compared with L_1, an approximate value of L can be obtained from (5)

$$L \approx L_1 + \frac{L^2_2}{2L_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The error in this value of L is less than $\frac{L^4_2}{8L^3_1}$, which is very small if $\frac{L_2}{L_1}$ is small.

If the angle between AB and the axis (1) is now further reduced the correction term $\frac{L^2_2}{2L_1}$ in (5) will decrease, and we obtain

$$L \approx L_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The error in this value of L is less than $\frac{L^2_2}{2L_1}$.

It is now seen that a variation in the accuracy of the measurement of L_1 and L_2 from one case to another may influence the choice of formula. If E denotes the error,

formula (4) is used if $E < \frac{L^4_2}{8L^3}$;

formula (5) is used if $\frac{L^2_2}{2L_1} > E < \frac{L^4_2}{8L^3}$;

and formula (6) is used if $E > \frac{L^2_2}{2L_1}$.

For a given required accuracy in the final result the permissible error of measurement may be determined. It may then be possible to use the simpler formulae (5) or (6) instead of (4), also in cases where the accuracy of measurement permits the use of the more rigorous formula (4).

Similarly the question of accuracy can play a role in mensuration in general. The error in the final result is often divided into two parts: one dependent on the inaccuracy in the actual measurements (the observations), the other dependent on the approximations in the calculation. This may suggest a convenient way of exploiting certain instruments for mensuration and also how the method should be modified to reduce the error. Just as for the measurement of the distance AB by various simplifications, different types of simplified procedures are indicated in stereo-radiography. We may either start with simplified procedures and evolve the method directly, or start from the general case and introduce approximations. In both cases the comparison with the general case suggests simplifications that may be introduced without prejudicing the final result. In 1951, *Berghagen* and *Hjelmström* studied the accuracy of intra-oral radiography, especially the error associated with the properties of a new apparatus and the evolution of the mensuration technique.

AN APPARATUS FOR INTRA-ORAL RADIOGRAPHY

Design

The design of an apparatus for intra-oral radiography should be based on the following principles:

(1) The film should be plane, to facilitate the treatment of the radiograph.

(2) The inner orientation must be known.

(3) The outer orientation must be reconstructable for single radiographs.

(4) The relative orientation and the length of the base should be known for stereo-pairs.

Of these (1) may be obtained by using a plane film-holder. The inner orientation should be determinable by measurement on one occasion, and this should serve for all subsequent exposures. It is then necessary to have a firm mechanical connection between the film-holder and the focus.

For the outer orientation to be reconstructable, the film must be replaced in an identical position on each exposure. This may be effected by means of an impression of the teeth, taken in impression compound affixed to the film-holder.

Since 1951 the method has been used clinically with the idea of simplifying it and at the same time increasing its accuracy with a view to performing measurements on the actual radiographs. Certain improvements in the design are described here, with comments on their value.

Description of the apparatus (Fig. 11, Fig. 12, and Plate 1)

The complete unit may be said to consist of two principal components: the film-holder and a clamping device by which the holder is fixed to the X-ray machine in the desired position with regard to the central ray.

The clamping device consists of a frame (1) cast in light metal, in which is fitted a thin steel cylinder (4); inside this a short tube (5) is loosely fitted, having two studs corresponding to the bayonet attachment on the X-ray machine. On the other end of the tube is threaded a knurled ring (8) by which the unit is securely fastened to the X-ray tube. The cylinder contains a lead diaphragm (5 a).

The film-holder unit, consisting of the holder itself (18), the impression-compound carrier (16) and the bracket (19), is of metal plate. It is fixed to the clamping device through a metal rod (3) of circular section — the film-holder arm. The arm has a cylindrical crosspiece (13) to which the bracket is secured by a bolt and a set screw (17 and 15). The head of the bolt is in the form of a disc bearing an engraved scale by means of which the film-holder unit may be rotated a known angle.

The opposite end (10) of the rod is square in section and is clamped in one of seven horizontal grooves (9) by the bridge (11) and two threaded screws (12).

The film-holder of metal plate is of a suitable size to take the film to be held in the mouth. The lateral edges of the plate are bent to form a frame in which the film may be inserted and held firmly. The edges serve as reference lines on the radiographs, and have notches — "frame marks" — as reference points. The

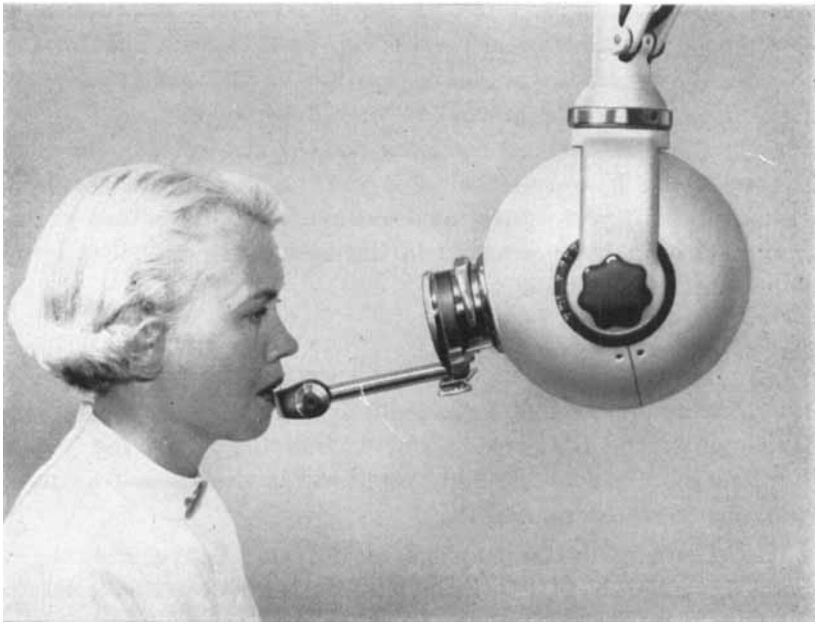

Plate 1. The new apparatus in clinical use.

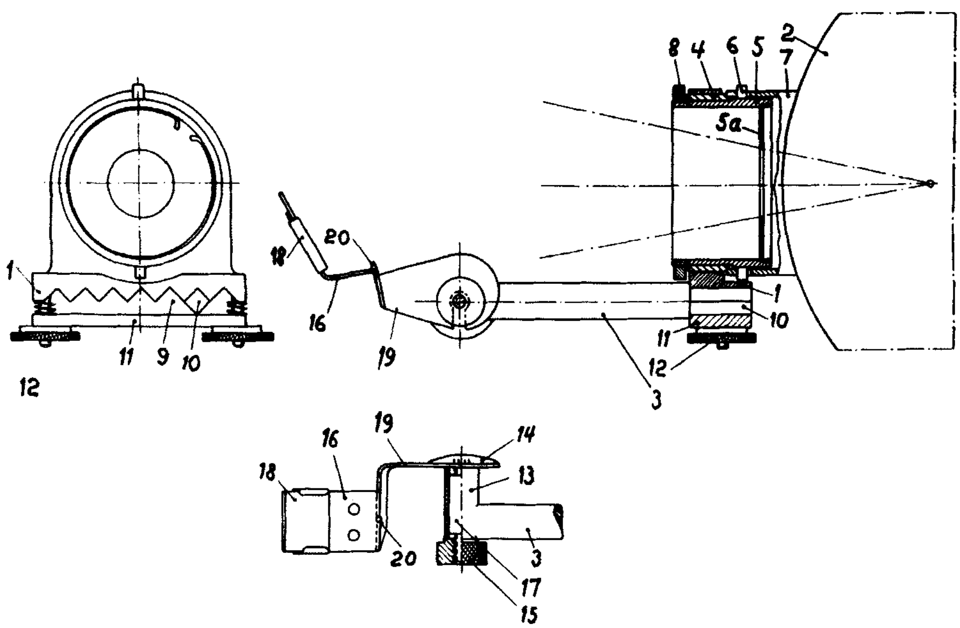

Fig. 11.

film-holder (18) is set at an obtuse angle to the impression compound carrier (19), the compound being placed on the upper and lower surfaces.

When the tray, film and impression compound are introduced into the mouth, the patient bites on the plate and thus provides an impression of the teeth that will enable the tray to be re-set in the same position after the film has been changed.

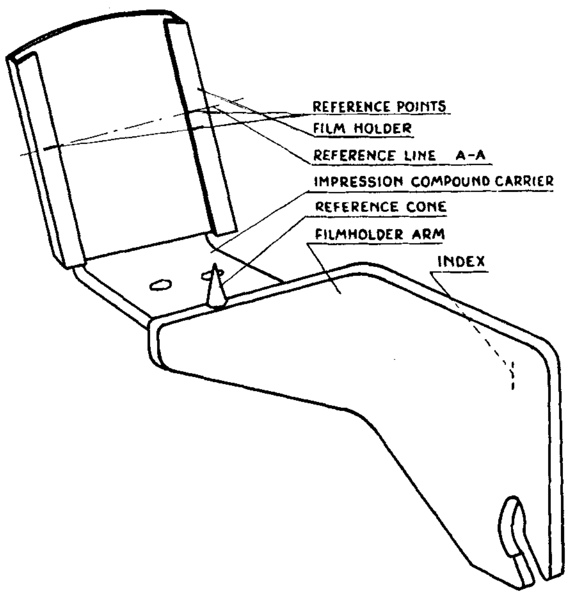

Fig. 12.

The bracket is bent at right angles to the impression-compound carrier. On the bracket, a small cone (20) — the reference cone — is fastened centrally in relation to the film-holder. The film-holder bracket is bent either to the right or to the left, so that the compound carrier may be used in the lateral regions of the mouth. On the left, the tray covers the +3 to 8+ and 3— to —8, while the corresponding teeth on the right are reached by the tray with a right bend. This film-holder unit may be rotated about the bolt (17) and can be locked by tightening the screw (15). The angle can be read on the engraved scale; the tray is easily removable.

Properties of the apparatus (Figs. 13 and 14).

The film-holder is designed for fixing the film in relation to the tooth or the part to be radiographed. If it is locked to the cylinder at a known angle, and the film-holder arm is in turn

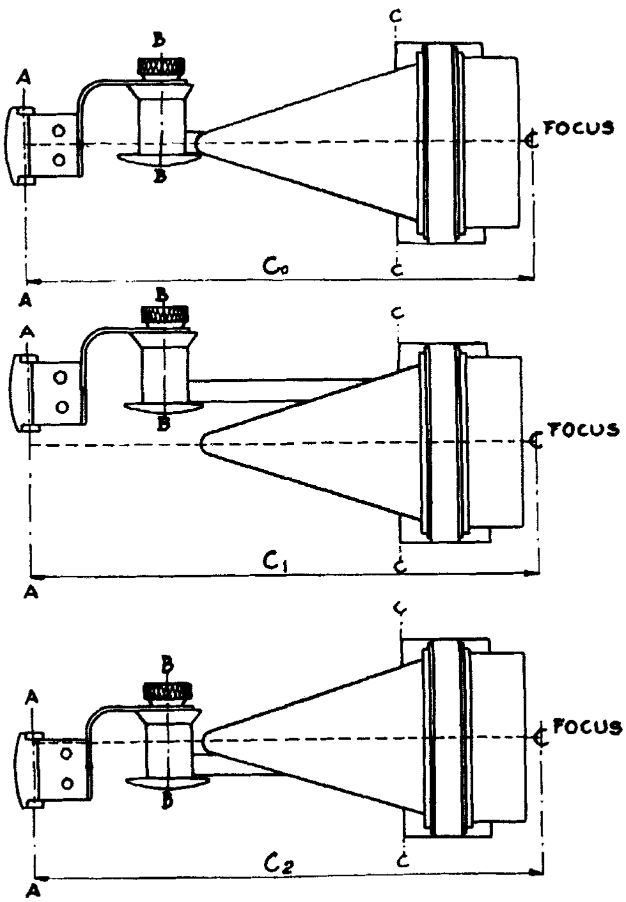

Fig. 13.

locked in one of the holes in the clamp, the film, object and focus will be set in a known relation to one another. The impression-compound carrier can be stored, and if the adjustment is noted it may subsequently be set up with fair accuracy in the same relative position. The holder ensures a plane film, so that

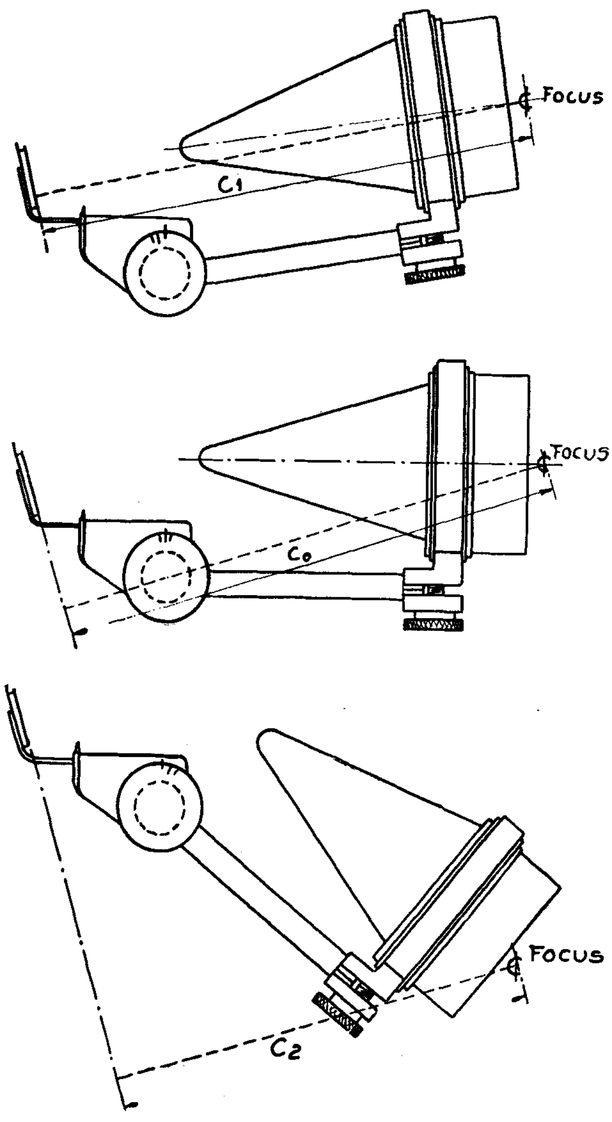

Fig. 14.

the whole image surface is available for study. It is further possible to determine accurately the region to be projected. The tube focus may be set in a variety of known positions, so that the part of the jaw to be radiographed may be brought into the field of projection. The clamp, with its seven grooves 10 mm apart, provides seven positions of the focus, the maximum base being 60 mm. This movement is parallel to the frame reference lines, thus maintaining a constant principal distance*. The tube focus may be rotated about the axis of the crosspiece independently of the selection of the base holes. The calibration of the disc is symmetrical about a zero mark, and each division represents a certain principal distance (Fig. 14). By moving from one division to the corresponding one on the other side of the zero, the focus is displaced along an arc whose ends are equidistant from the film plane. The chord is the base of the stereo-pair, and with reading +n and —n has the length 2n.

To obtain a stereo-pair two radiographs are taken, the focus being shifted a certain base distance between the exposures. This alters the inner orientation, and the change may be determined in magnitude and direction. For simplicity of mathematical treatment, the same principal distance should be used in both exposures. As already mentioned, the principal distance is not changed by moving the film-holder arm from one groove to another. For each adjustment of the joint there is a symmetrical position with unchanged principal distance. This system provides fourteen positions of the tube focus with constant principal distance. The practical application of this choice of focus position is evident from the following example:

Let us suppose a radiographic examination of an upper left first molar and the surrounding tissues is to be performed. A stereo-pair is taken with cylinder positions +2 and —2, and with the base arm in hole IV. It is found that on one of the radiographs (position —2) the malar bone overshadows the apical region of the tooth, but that the second of the pair (position +2) is acceptable. The three-dimensional effect can be obtained only at points that are clearly reproduced on both radiographs. If the structure of the apical region is of interest it is best to use +2 as the basic position and to obtain the stereo-pair by choosing two positions for the film-holder arm.

A further example: let us suppose that only a part of the crown of an impacted canine has been projected on the films when the cylinder readings were +2 and —2 and the centre

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* Principal distance = distance from focus to film plane.

hole in the base plate has been used. To get the whole crown on to the film it is unnecessary to change the relative positions of the film and the crown by making a fresh impression. Instead, the focus may be moved by selecting some other base hole, having due regard to the position of the crown in the first pair. The two exposures are then made with the original rotary settings +2 and —2.

From a study of a preliminary radiograph, any required change in perspective may be obtained by changing either the rotary or horizontal setting.

The purpose of the reference cone is to provide a check on the inner orientation for each radiograph of a pair. It is also possible to see whether the radiographs have been arranged correctly. If the two are interchanged a pseudoeffect is obtained, the relief and depressions being reversed in the stereo-image.

Application in practice

Some 2,600 intra-oral radiographs have been studied with respect to the improved diagnostic facilities offered by this radiographic technique, especially in oral surgery.

This method would appear to meet the demand for a more reliable radiographic technique to be applied both for single radiographs and stereo-pairs. Moreover, it provides the scientific worker with a method of performing measurements on the actual radiographs.

Dental intra-oral radiographs are used chiefly for diagnosis, planning an operation and for studying the results of treatment. In many cases the diagnosis can be made without considering the geometric properties of the radiograph, but quite often this is desirable or even necessary. The isometric rule for the adjustment of the central ray has long been followed. The ray is generally aimed at the invisible root tip and perpendicularly to the bisector of the angle between the long axis of the tooth and the film plane. It is never possible to know whether the requirements of the isometry rule have been fulfilled.

The method described here is a systematic procedure using known factors only, so that photogrammetrically correct radiographs are obtained rapidly and easily, permitting a radiographic examination of a three-dimensional object of odontological interest. Fourteen variations of the radiographic image are available without changing the impression compound, and the radiographs may be studied either as separate pictures or as stereo-pairs. This considerably increases the possibility of accurately diagnosing a pathologic condition, since superimposed details may be separated in the stereo-model, with improvement in definition.

With a knowledge of the features of the system one can employ it in radiological research.

If great accuracy is required in the evaluation of the results of treatment by viewing or mensuration on the radiograph, stereo-pairs may be taken and the stereo-models examined. Later on, more radiographs are taken with roughly the same settings. The new stereo-models are then to be treated in the same way and comparisons made with the previous results. This is possible so long as the apparatus is calibrated — particularly with respect to the position of the focus. The radiographs are unfortunately not true reproductions of the object but only a record of its mass. If the base is suitably chosen, a stereo-pair may be obtained in which measurement may be made of details visible in both radiographs. By using the shortest base (10 mm with the apparatus described) very good definition is obtainable. It is also easier to recognize object details on the two separate films. The accuracy is diminished with a shorter base, but this is compensated by the increased ease in recognizing corresponding measuring points. By treating such stereo-pairs it is possible to make three-dimensional examinations. The apparatus can be used without the impression compound, for routine examination. It still has the advantages that the film is plane and that other conditions are stable during the exposure.

Single radiographs may be viewed in the direction of the rays or against them. Which method is correct? According to the rules of perspective there is only one position of the eyes. Viewing as from the mirror image of this position gives a mirror reversal of the object viewed, since all points in the image are reflected in themselves. The effect of depth that is obtained will, on the other hand, be unchanged. It should be noted, however, that the perspective on viewing single pictures can give the correct effect of depth only if the shape of the reproduced object is known. The radiographic image perspective, in other words, does not provide a means of judging the shape of unknown three-dimensional figures if only a single radiograph is used. There are four ways of combining the radiographs for studying the stereo-models. Only the one reproducing the inner orientation of both pictures simultaneously gives the correct perspective (Plate 2). If the radiographs in this position are viewed from the opposite side, a mirror-image space-model is obtained (Plate 3). If, on the other hand, the radiographs are interchanged a pseudoscopic effect is obtained — that is, the foreground and background change places (Plate 4). Seen from behind, such pairs give a pseudoscopic reversed image (Plate 5). Such models are not correct in shape. The two former stereo-models (with orthoscopic effect) are correct in shape and mirror images of each other. With the orthoscopic effect the films may be viewed either in or against the direction of the rays, the only difference being the mirror reversal reflection. On some occasions it might be advantageous to make use of the pseudoscopic effect, since the definition is increased and it is easier to decide where in space certain details are situated. The discrepancy in shape in such cases is not disturbing. For example, consider the case of a vertical pocket situated proximally in the bone, this pocket being open on the lingual but closed on the vestibular side. The three-dimensional conditions will be better recognized by viewing the radiographs from the lingual side, i.e., against the ray direction.

To sum up: the experiments have resulted in a method that is of assistance in the clinic. The systematic procedure using only known factors would seem to eliminate the uncertainties to which intra-oral radiography by the "free technique" is subject.

When moreover, it is easy to obtain such radiographs the method is to be recommended. It is clear that the technique is worth utilizing if only for the possibility of performing measurements on the film, and with no need for stereoscopic viewing.

SUMMARY

The principles of intra-oral radiography are outlined. A discussion of the possibilities of performing measurements on the radiographs is followed by a description of a new method and the construction of a new apparatus for intra-oral radiography.

The advantages of the new method are:

1. the radiographic procedure is simple.
2. it is advantageous to view in three dimensions.
3. the quality of the radiographs is superior to those taken by the "free technique" (e.g. there is no movement during exposure).
4. accurate measurements may be performed on the radiographs.

Experience gained in the clinical application of the method is touched upon, and it is shown that the experiments have resulted in a method that may be of value in practice. The systematic procedure, using only known factors, should eliminate the uncertainty inherent in the "free technique".

RÉSUMÉ

LE RADIOGRAPHIE INTRA-ORALE EXPLIQUÉE PAR LA GÉOMÉTRIE DANS L'ESPACE

Sont exposés tout d'abord les principes théoriques de la radiographie intra-orale. Les diverses possibilités d'effectuer des mesures sur la radiographie sont examinées. Exposé d'une technique nouvelle de radiographie intra-orale et description d'un appareil nouveau.

Les avantages de cette nouvelle méthode sont les suivants:

1. La manière de procéder est simple.
2. Les radios stéréoscopiques ainsi obtenues se prêtent mieux à l'examen que des images simples.
3. L'image est de meilleure qualité que dans le cas de radios prises selon la "technique libre" (l'appareil étant fixe, l'image n'est jamais floue).
4. Les radios se prêtent à des mesures précises.

Enfin les résultats acquis par l'expérience clinique montrent que les recherches ont abouti à un procédé susceptible d'être utilisé dans la pratique. La méthode systématique, faisant usage exclusivement de facteurs connus, proposée dans cette étude intra-orale éliminerait complètement le facteur d'incertitude toujours présent dans le cas de radios obtenues par la "technique libre".

ZUSAMMENFASSUNG

DIE INTRAORALE RÖNTGENAUFNAHME IM LICHTE DER DREIDIMEN SIONELLEN GEOMETRISCHEN BESCHREIBUNG

Zunächst wird die theoretische Grundlage der intraoralen Röntgenaufnahme besprochen. Die Möglichkeiten auf Röntgenbildern Messungen vorzunehmen werden diskutiert. Eine neue Methode der intraoralen Röntgenaufnahme und eine neue Apparatur werden beschrieben.

Die Vorteile der neuen Methode sind folgende:

1. Die Aufnahmetechnik ist einfach.
2. Stereoskopische Röntgenaufnahmen können angefertigt werden. Solche Aufnahmen sind zur Auswertung besser geeignet als Einzelbilder.
3. Die Bildqualität ist besser als bei Aufnahmen, die nach der "freien Methode" angefertigt werden. (Keine Bewegungsunschärfe.)
4. Die Aufnahmen können für Messungen benützt werden.

Zum Schluss werden die klinischen Erfahrungen berührt und es wird gezeigt, dass die Versuche zu einer Methode führten, die sicherlich in der praktischen Arbeit von Nutzen sein kann. Die systhematische Verwendung ausschliesslich bekannter Faktoren, wie es hier für die intraoralen Röntgenaufnahmen vorgeschlagen wird, scheint die Unsicherheit, die bei der Anfertigung von Aufnahmen nach der "freien Methode" vorhanden ist, vollkommen beseitigen zu können.

RESUMEN

PRINCIPIOS FUNDAMENTALES DE LA RADIOGRAFÍA TRIDIMENSIONAL INTRAORAL

Inicialmente se exponen los principios teóricos de la radiografía intraoral. Se examinan las diferentes posibilidades de

llevar a cabo medidas en las radiografias. Descripción de un nuevo aparato y de una nueva técnica para tomar radiografías intraorales.

Las ventajas de este nuevo método son:

1. La técnica radiográfica es fácil.
2. La observación en tres dimensiones es muy ventajosa.
3. La calidad de las radiografías es superior a la de aquellas tomadas con la "t'ecnica libre" (por ejemplo: no hay movimiento durante la exposición).
4. Las radiografías se prestan a medidas precisas.

Los resultados obtenidos por la experiencia clínica muestran que las investigaciones han llegado e establecer un método que puede ser de valor en la práctica. El método sistemático, haciendo solamente uso de factores conocidos, eliminaria la incertidumbre siempre presente en el caso de radiografías obtenidas con la "técnica libre".

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