From: The Roentgen Department (Director: Yrjö V. Paatero, Docent, Dr. Odont.) of the Institute of Dentistry, University of Helsinki.

ON THE SHAPE OF VERTICAL SURFACES IN STEREO-PANTOMOGRAPHY*

by

Yrjö V. Paatero

According to the basic principle of pantomography, a sharp image is projected upon a linearly moving or rotating film only of that surface of a rotating object which moves through a narrow roentgen beam at the same linear speed as the film, the said surface being at rest in relation to the film. In practice, however, the objects in the immediate proximity of the surface being reproduced in sharp focus also show up so clearly, on account of the negligible difference in speed, that the image in a pantogram always, in fact, represents a thin layer and not a mathematical surface. It follows from this that it is possible to apply stereoscopy, too, to pantomography, as demonstrated in many earlier papers by the present author [3, 5, 6, 7]. Besides the depth of the layer reproduced, a stereopantomogram usually also brings out the curvature of the layer. This does not in itself, however, result from the fact that the layer is curved, and the curvature in the picture does not generally correspond in form to the curvature of the layer reproduced. This phenomenon in connection with radiography of the jaws[6] was described in a preliminary manner by the author as early as in 1954. In the present paper the author will deal with the dependence of the *form* of a stereoscopically radiographed layer, as visible in the picture, or, more accurately stated, the surface, on the form of the actual surface, leaving the depth of the layer out of consideration. In this connection the author shall limit himself to vertical surfaces. Oblique and convex surfaces will have to await later treatment. He shall also ignore, for the sake of clarity, paraboliform and hyperboliform vertical surfaces as well as the various combinations,

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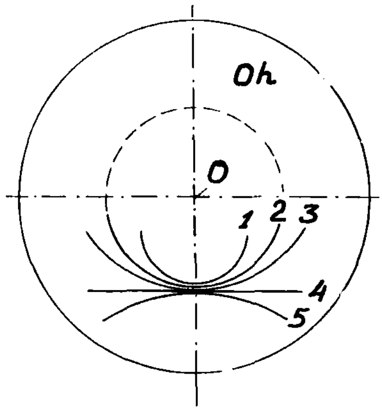


Fig. 1. Grouping of pantomographically reproduced vertical surfaces.

and shall content himself with discussing flat planes and circular arcs.

As for the technique of radiography, the author is confining himself chiefly to the use of a rotating cassette both in theoretical examination and in practical tests. In order to achieve a stereoscopic effect, a vertical tube shift on a 5 cm base was exclusively used.

Within this scope vertical surfaces may be divided into five groups as illustrated by Fig. 1. The first group consists of the curved surfaces that lie tangent to some concentric circumference of the object holder (Oh) or, in practice, touch a concentric cylindrical surface at one point, or with a line, otherwise being inside the said perimeter. In other words, the center of the radius of the arc of these surfaces is somewhere between the said point of tangency and the rotational axis (O) of the object holder. This group of surfaces is represented in Fig. 1 by the arc marked no. 1.

The second group consists of surfaces situated concentric with the axis of the object holder. The center of the curvature radius of these surfaces is situated, of course, on rotational axis O, like that of arc 2.

The surfaces of the third group lie tangent to some concentric circle at one point, being otherwise outside it, like arc 3. The center of the curvature radius of these surfaces is on the side of the rotational axis opposite to the point of tangency.

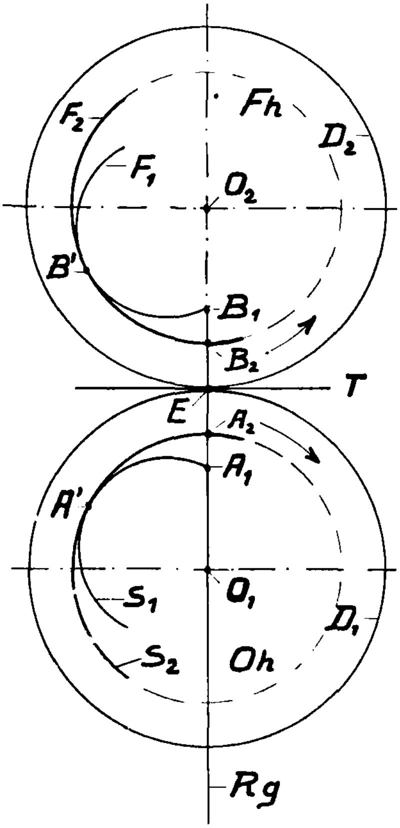
The surfaces of the fourth group are straight planes, which, of course, lie tangent to some concentric circle at one point, like the straight line 4.

The surfaces of the fifth group are situated outside one of the concentric circles and touch at but a single point, which is between the rotational axis (O) and the center of the curvature radius of the said surface. Arc 5 illustrates this group of surfaces.

All the surface types may lie or may be placed, within certain limits, at any distance from the rotational axis. Likewise, the radii of their curvatures may be longer or shorter than in Fig. 1 and the length of the surfaces may extend all the way to the outer circumference of the object holder.

If a beam comprising parallel X-rays[4], were available the film could be bent exactly into the shape of the surface to be radiographed and placed on the film holder in a position corresponding to that surface. In that case Fig. 1 would represent the film holder and the films placed on it. Since, in practice, however, the rays from present-day roentgen tubes start from a pointlike focus and then diverge in forming a beam, a so-called divergency rectification[5] must be taken into account in placing the film. This means that the film has to be placed slightly farther from its axis than the surface being radiographed is from its axis. The divergency rectification diminishes toward the outer circumference, being, of course, non-existent there, for the surface being reproduced and the film are in contact. Therefore no enlargement of the picture, which would expressly require a divergency rectification in pantomography, takes place.

Since the divergency rectification does not essentially affect the shape of the surface visible in the stereoscopic image, we shall ignore it in this connection, although it does affect the magnitude of the stereoscopic effect, which anybody may observe for himself by means of the formula for a stereoscopic effect presented by *Mannila*.[1] The purpose of the author is only to show in what direction a surface of any particular shape bends in a stereoscopic projection but not to calculate how much it bends. In other words, this study is qualitative rather than quantitative. Accordingly, a film may be placed so as to correspond entirely to the surface projected which will simplify and clarify the treatment of the matter. This has been the procedure observed in



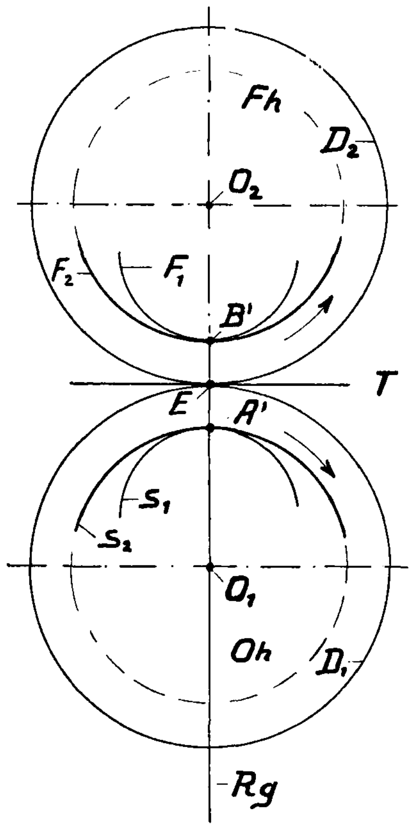


Fig. 2. Surfaces of the first and second groups during the initial stage of rotation.

Fig. 3. Surfaces of the first and second groups during the middle stage of rotation.

respect to the following illustrations, by means of which we shall now examine in greater detail each of the groups of surfaces mentioned earlier.

The surfaces of the first and second groups are illustrated schematically in Figs. 2 and 3. In the former, object holder Oh rotates about its axis O_1 and film holder Fh around its axis O_2. Cylindrical films F_1 and F_2 are placed on the film holder, F_1 lying tangent to one of the concentric circles at point B'. The roentgen beam (Rg) is directed via axis O_1 toward axis O_2. Since holders Oh and Fh rotate at the same angle speed, an accurate image is projected on the films of the corresponding surfaces

S_1 and S_2 of the object by virtue of the fact that they move in pairs at the same linear speed through the roentgen beam as well as in the same direction, as indicated by the arrows. (If a film moving along a straight course were used, it could be suitably moved along tangent T and film holder Fh would be unnecessary). The peripheries D_1 and D_2 of the holders touch each other at point E with the common tangent T. When film F_1 has reached the roentgen beam at point B_1, a continuous image of surface S_1 begins to be projected on it. The corresponding point of S_1 is A_1. As the rotational movement continues, the point of intersection B_1 of the roentgen beam and the film approaches the circumference of the circle along which point B' moves, until B_1 joins point B' (Fig. 3). Thereafter, point B_1 again begins to approach rotational axis O_2. We may observe that point B_1 is always nearer to axis O_2 than is point B', except when they have joined. From this follows that $EB_1 > EB'$. Quite the same observation is to be made in regard to surface S_1, $A_1E > A'E$. Further: $A_1E + EB_1 > A'E + EB'$. From this it becomes evident that the surface projected and the film first approach each other and then, when points A' and B' have passed the roentgen beam, once more move apart. If the stereopantomogram is viewed from the side at which the roentgen tube was situated during the exposure, surface S_1 appears to be concave toward the observer, for point A' is farthest from him. If, instead of on the cylindrical film here presented, the picture were taken on a straight film while it moved at the distance of tangent T from the object, the curvature of the surface in the stereopantomogram would be only half of what it is with a cylindrical film, since the latter approaches and draws away from the tangent during exposure, just as does the projected surface.

The author checked the validity of these considerations by means of the following phantom test: A screen was bent into an arc with a radius of 5 cm and placed onto the object holder in such a way that the center (point A') of the screen was at a distance of 8 cm from rotational axis O_1. A film contained in a flexible plastic cassette was placed on the film holder in the corresponding manner, taking into account the divergency rectification. On the surface of the screen toward the film was placed the letter "L", made of lead, to indicate the convex side. On the surface of the cassette several lead pellets were placed for the purpose of indicating the situation of the film surface in the stereoscopic picture. Since the tube was shifted vertically, as pointed out before, the picture components should lie on their sides (Fig. 4). In looking parallelly at the pictures, we see that the concave side of the screen is toward us; but when viewed with crossed eyes, it is away from us. This experiment thus led to the same result as our theoretical study. The same experiment was performed with screens bent at varying degrees, the results constantly being the same. We further note that the pellets are in a straight plane, provided the picture or this paper is not bent.

The other film F_2 and the surface S_2 projected onto it, drawn in Figs. 2 and 3, are arcs, the centers of which are situated on the corresponding rotational axes O_2 and O_1. Hence, every point on the film and the projected surface is equidistant from its rotational axis. Accordingly, all the points of the arc in question are also located at the same distance from the outer rims of the holders. Thus $A_1E = A'E$ and $EB_2 = EB'$. The distance between the projected surface and the film remains constant throughout the entire rotation and exposure. Considering, moreover, the fact that both the distance of the projected surface and that of the film from the focus of the roentgen tube remain unchanged while a continuous image is being projected onto the film, it is to be expected that the picture obtained stereoscopically of such a cylindrical surface cannot appear curved in a three-dimensional pantomogram but must appear to be an altogether straight surface. The curvature of the projected surface cannot in itself, therefore, bring about the curved surface observed in the picture, but this is caused by the eccentricity of the center of curvature of the projected surface from the rotational axis.

The phantom test carried out with a screen proved the matter to be as described above. In this case, too, the author bent the screen into a cylindrical form and placed it onto the object holder. This time, however, it was bent in such a way that the center of curvature of the screen was on axis O_1. The radius of the curve of the screen was 8 cm. The result is seen in Fig. 5. The screen appears to be straight, although it was curved during the exposure. The slight waviness in the picture results from

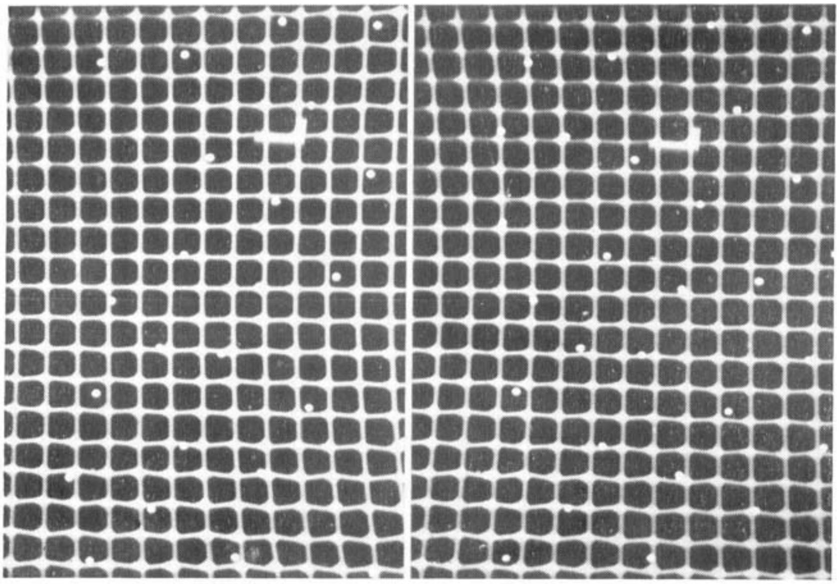


Fig. 4. Stereopantomogram of the screen corresponding to the surface of the first group.

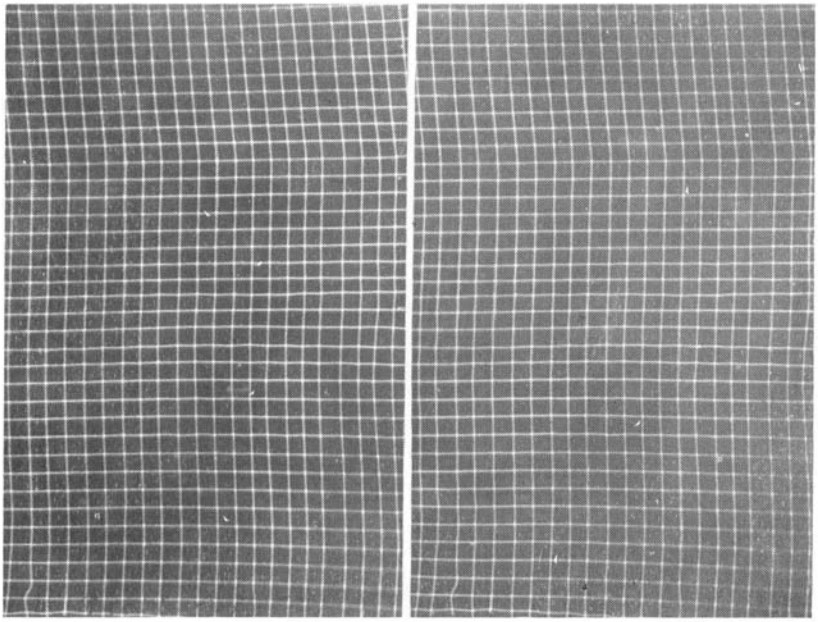
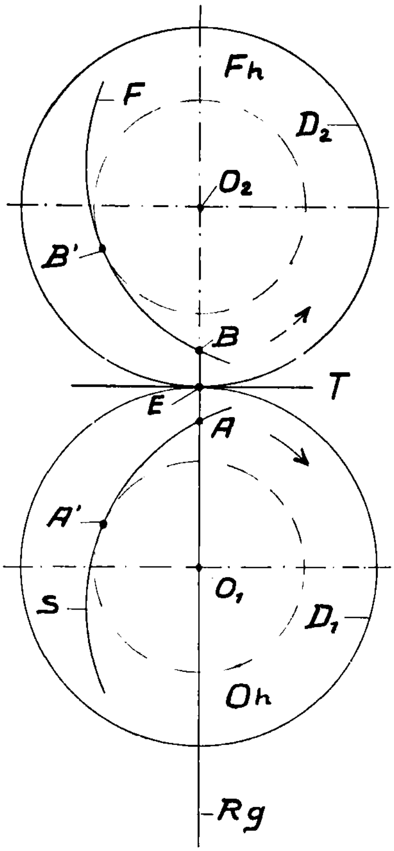


Fig. 5. Stereopantomogram of the screen corresponding to the surface of the second group.



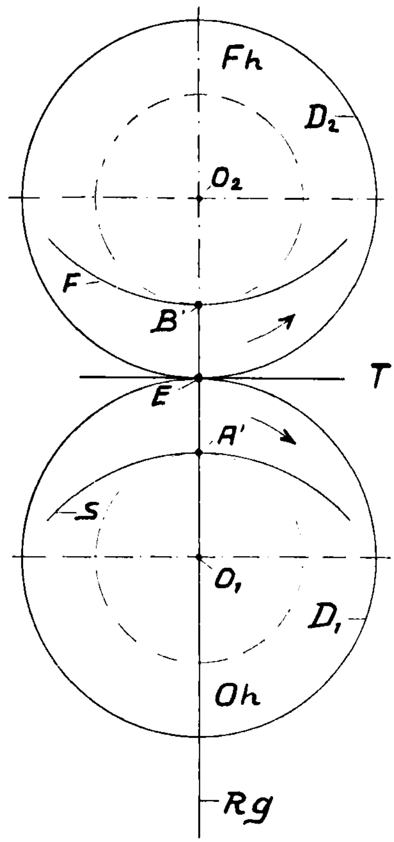
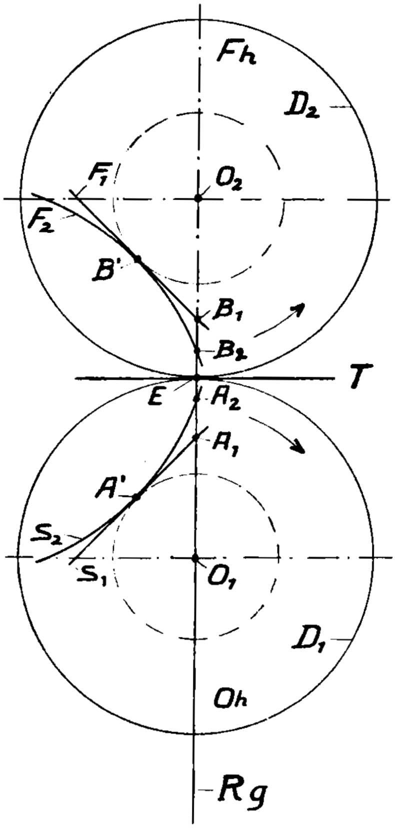


Fig. 6. Surface of the third group during the initial stage of rotation.
Fig. 7. Surface of the third group during the middle stage of rotation.

the fact that it was impossible to bend the fairly stiff screen absolutely evenly by hand. Some small waves remained, which naturally are visible in the stereoscopic picture.

Figs. 6 and 7 represent the curved surface (S) of the third group, that is, a case where the projected surface lies tangent to a concentric circle of the object holder (at point A′), and is outside this circle. The center of the radius of the curvature of the surface in this case is thus on the opposite side of the rotational axis (O_1). The point of tangency of the film is B′ and the center of the radius of its curvature "behind" axis O_2. The illustrations show that $AE < A'E$ and $EB < EB'$. It follows that



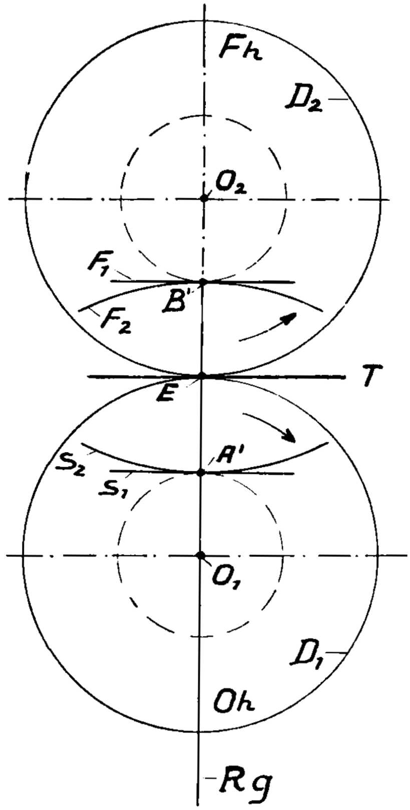


Fig. 8. Surfaces of the fourth and fifth groups during the initial stage of rotation.

Fig. 9. Surfaces of the fourth and fifth groups during the middle stage of rotation.

$AE + EB < A'E + EB'$, which means that the center of the surface is closer to the roentgen tube than the other parts. Viewing the surface in the stereopantomogram parallelly from the side of the roentgen tube, the surface must accordingly appear convex (in a horizontal direction). The test carried out with the screen led to the same result. Since the stereoscopic picture is almost identical with the one taken of a straight screen (Fig. 10), it is not published here.

In Figs. 8 and 9 there are again two surfaces, representing the fourth and fifth groups. One of the surfaces (S_1) is flat, the other (S_2) is curved, with the concave side outward. The corresponding films are F_1 and F_2. The points of tangency are, as before, A′ and B′. The surfaces intersect the roentgen beam at points A_1 and A_2, and the films at points B_1 and B_2 (Fig. 8). During rotation the points of intersection at first move closer to the axes, the minimum being at points A′ and B′ (Fig. 9) and then, once again, closer to the peripheries (D_1 and D_2). If we first examine the straight surface S_1 and the film F_1 associated with it, we note that $A_1E + EB_1 < A'E + EB'$, from which we may judge the straight surface to appear convex when viewed parallelly from the side of the roentgen tube, for the center of the surface will be closest to the observer. As for the shape of surface S_2, this too appears convex when viewed parallelly from the side of the roentgen tube, for $A_2E + EB_2 < A'E + EB'$.

Fig. 10 presents the result of the phantom verification test: Viewed parallelly the straight screen appears convex, in agreement with the result obtained geometrically. The lead letter "L" was on the side of the screen facing the film during exposure.

Bent to correspond to the surface of the fifth group, the screen likewise looks convex when viewed parallelly from the side of the roentgen tube, as may be seen in the stereopantomogram of Fig. 11. In this case, too, the phantom test proved that the geometric study led to the correct result. The side areas of the image have contracted considerably, for in this instance the roentgen beam travels very obliquely through the surface except at the center.

A stereoscopic picture of a dry mandible is presented in Fig. 12. In pantomographing this mandible an ordinary bendable cassette was used. The front part of the cassette — corresponding to the shape of the jaw — formed a regular arc, while both sides were straight. On the surface of the side of the cassette toward the roentgen tube was placed a celluloid sheet with lead pellets to show the film surface during examination of the stereopantomogram. In addition, a horizontal strip of lead shield was placed between the object and the film, across its vertical slit. By casting a white, bandlike shadow across the entire picture, this simultaneously reveals the bend of the surface in the event that, instead of the bent cassette, a straight one is used which moves along the joint tangent (marked T in the figures) of the object and film holders.

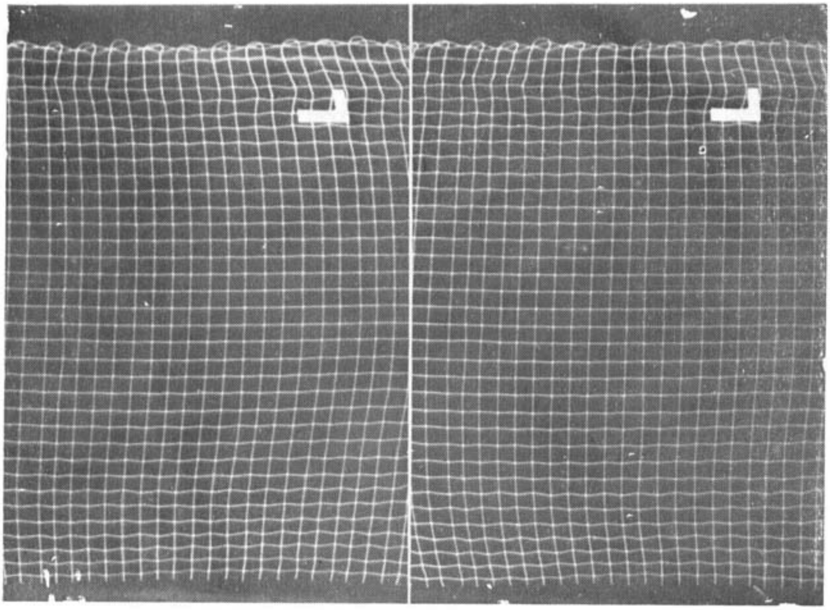


Fig. 10. Stereopantomogram of the screen corresponding to the surface of the fourth group.

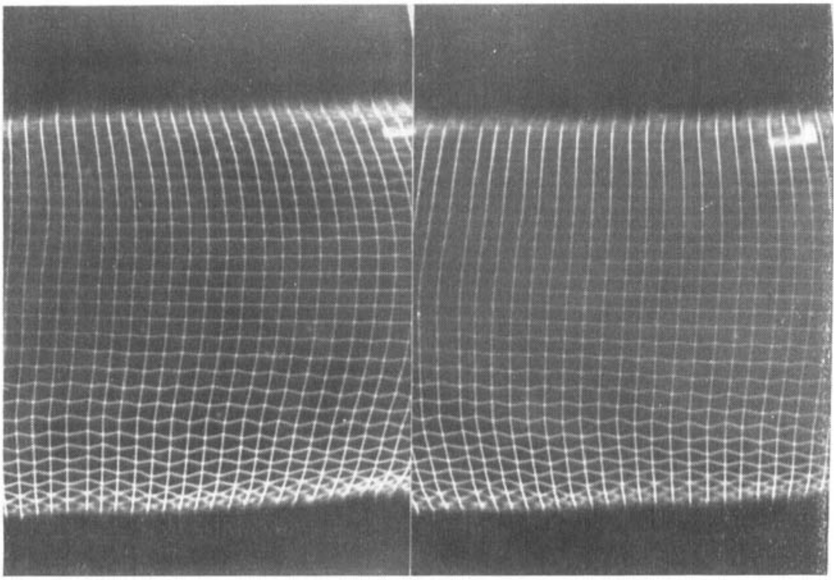


Fig. 11. Stereopantomogram of the screen corresponding to the surface of the fifth group.

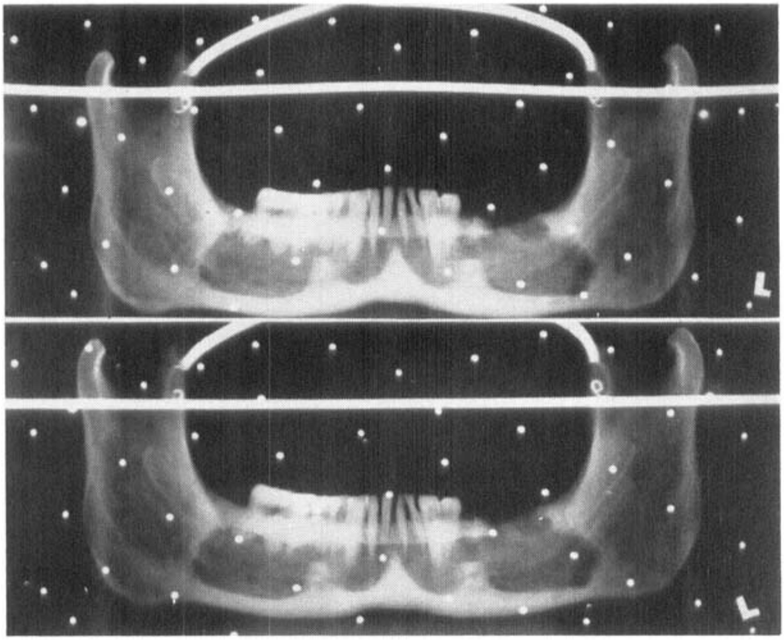


Fig. 12. Stereopantomogram of mandible.

When Fig. 12 is examined parallelly, i.e. from the side of the roentgen tube, the curved anterior part of the jaw appears to be concave, for it belongs to the first group of surfaces, the radius of which is smaller than the radius of its concentric circle, which the surface lies tangent to from the inside. We thus see the jaw as if from the back of the neck. Looking crosswise, we see the jaw from the outside. Since the sides of the film were straight, the lateral portions of the mandible projected on them belong among the surfaces of the fourth group, which in the stereopantomogram curve toward the observer (toward the roentgen tube). The film surface indicated by the lead pellets (here the surface of the paper) greatly facilitates the perception of curves in different directions. The waviness of the shadow "drawn" by the strip of lead in the slit of the shield is beautifully brought out in the stereopantomogram, although its curvature is only about half the corresponding effect achieved by the mandible. This phenomenon has previously been elucidated by the author[6] and therefore is not dealt with in the present paper.

SUMMARY

A geometric and experimental study has been made with the object of determining the form in which vertical surfaces of given shapes appear in a stereoscopic pantomogram. The following observations were made.

When looking at the stereo-pantomogram parallelly from the side where the roentgen tube was during exposure, a surface appears concave to the observer only when the curve radius of the projected surface was shorter than the radius of a hypothetical cylindrical surface, tangent to the former and concentric with the rotation axis of the object holder. If, however, the curve radius of the projected surface was longer than the radius of the cylindrical surface mentioned, or if the surface projected was a straight plane or convex towards the rotation axis, then it appears convex to the observer in the stereo-pantomogram under the conditions described. When the centre of curvature of the projected surface was on the rotation axis, the surface has the appearance of a straight plane.

RESUME

SUR LES FORMES DES SURFACES VERTICALES DANS LA STEREO-PANTOMOGRAPHIE

Une étude géométrique et expérimentale a été faite dans le but de déterminer les formes que les différentes surfaces verticales prennent dans un pantomogramme stéréoscopique.

En regardant le stéréo-pantomogramme du côté où se trouvait le tube Roentgen pendant la prise de l'image, la surface se présente concave à l'observateur seulement si le rayon de la surface projetée est plus court que celui d'une surface cylindrique imaginaire, tangente à la précédente et ayant pour axe l'axe de rotation du support de l'objet. Si le rayon de la surface projetée est plus long que celui de la surface cylindrique susdite, ou bien, si la surface projetée est plane ou convexe vers l'axe de rotation, elle se montre convexe vers l'observateur dans le stéréo-pantomogramme considéré de la même manière que ci-dessus. Si l'axe de la surface projetée est congruent avec l'axe de rotation, la surface semble plane.

ZUSAMMENFASSUNG

ÜBER DIE FORMEN VON VERTIKALEN FLÄCHEN IN DER STEREOPANTOMOGRAPHIE

Der Verfasser hat sowohl geometrisch als auch experimentell untersucht, welche Form verschiedene vertikale Flächen in einem Stereopantomogramm annehmen.

Wenn das Stereopantomogramm parallel, von der Seite, wo sich die Röntgenröhre im Augenblick der Aufnahme befand, betrachtet wird, erscheint das Pantomogramm dem Betrachter konkav nur, wenn der Radius der aufgenommenen Fläche kürzer ist als der Radius einer angenommenen tangierenden, mit der Umdrehungsachse des Objekthalters konzentrischen zylindrischen Fläche. Wenn der Radius der aufgenommenen Fläche länger ist als der der obengenannten zylindrischen Fläche, oder wenn die aufgenommene Fläche plan oder konvex gegen die Umdrehungsachse ist, erscheint sie im Stereopantomogramm dem Betrachter konvex unter den oben beschriebenen Verhältnissen. Wenn der Radius der aufgenommenen Fläche und die Umdrehungsachse sich decken, erscheint die Fläche plan.

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Address: *Runeberginkatu 15 A, Helsinki*