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ON THE SHAPE OF CONICAL SURFACES IN STEREO-PANTOMOGRAPHY*

by

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The present paper represents a continuation of a paper[8] dealing with the shape of vertical surfaces in stereo-pantomography and is limited to the consideration of such conical surfaces as are most frequently met with in routine everyday pantomographic radiography.

As has previously been pointed out by the author[2—8] it is possible to project pantomographically surfaces of every shape and at all depths. In practice, these surfaces are thin layers. Pantomography of conical surfaces is quite simple, inasmuch as the ordinary roentgen film can always be cut and bent into the desired conical shape, thereby permitting the projection of a correspondingly shaped surface; and this is particularly the case when using pantomography, in which the rotating cassette technique is applicable, as illustrated by photographs published previously[5].

In addition to the depth of the projected layer, a stereoscopic pantomogram also brings out its shape, i.e., its possible curvature or inclination. Since the depth or thickness of the layer has no significance in this case, we shall devote our attention solely to the forms, and accordingly for the sake of preciseness, the author shall deal with surfaces and not layers. As a matter of fact, only a mathematical surface can be accurately projected onto a film, because the film can be stationary only in relation to a single surface in traveling at the same linear speed through a narrow vertical roentgen beam.

The conical surface to be reproduced — or, correspondingly, the film — may be the surface of either a taller or shorter cone

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or part of a cone, such as a truncated cone, or part of the surface of a cone cut parallel to its axis, which part when straightened out forms a sector of a circle. In addition, the axis of the conical surface (and also of the film) may be inclined to any degree and in any direction. If the axis is vertical, like the axes of the object holder and film holder, it may be situated either on these axes or eccentrically in relation to them. In the author's opinion, however, it is not necessary to consider all the existing possibilities. A few typical instances will suffice to demonstrate the dependence of the shape of a conical surface on certain factors in stereopantomography.

The author used a plastic vessel shaped like a truncated cone, with a diameter of 8 cm at its base and with an inclination of 50 degrees of the outer surface toward the axis. Alongside a semicircle he attached to its outer surface two horizontal rows of lead pellets as well as a few copper rings in order to indicate the location of the surface of the cone, since plastic does not show up in a roentgen picture. For the sake of uniformity the accompanying perspective diagrams show half a cone of the same shape, and for the sake of clarity they are drawn to full height so that the positions of the axes of the cones can be better seen in different cases. It is, of course, quite immaterial whether in depicting conical surfaces the apex is pointed down or up. In both cases the stereoscopic effect is the same, provided conditions are otherwise the same.

Let us now examine the conical surface in three different positions in relation to the rotational axis O_1 of object holder Oh. In Fig. 1 the projected surface is placed concentrically on object holder Oh, and the axis of the cone is identical with the rotational axis. The film is correspondingly situated on film holder Fh, bent into the shape of the surface to be projected. In this connection we leave the so-called divergency rectification required by the divergency out of account[4]. In practice, because of the divergency of roentgen rays, the film should be situated slightly farther from its rotational axis than the surface to be projected is from its axis, as has been explained in earlier papers[4, 5, 7, 8]. Since the object holder and the film holder rotate during exposure in opposite directions at the same angular speed while the narrow roentgen beam Rg is directed via axis O_1 toward

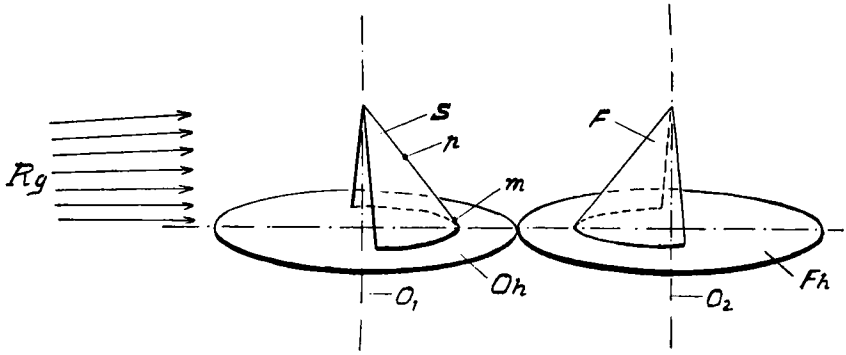

Fig. 1. Pantomographic roentgen projection onto conical film F of concentric conical surface S. As object holder Oh and film holder Fh rotate in opposite directions at the same angular speed, the narrow roentgen beam Rg is aimed via rotational axis O_1 toward rotational axis O_2. In stereoscopic pantomography two exposures are made on separate films from slightly different angles.

axis O_2, the image is projected according to the principle of pantomography from conical surface S to film F, which when straightened out takes the form of a sector of a circle. If another identically shaped film be exposed from a slightly different angle according to stereoscopic requirements[1, 5, 6, 7], two stereoscopic picture components are obtained, which when observed together correctly form a three-dimensional picture of the surface in question. In a stereoscopic picture viewed from the side of the roentgen tube the parts of the object which were nearest to the tube during exposure appear to be nearest the viewer and vice versa. The projected surface must also appear in the stereoscopic picture in such a way that it is inclined or bent in the same direction as it actually was. Thus, e.g., an arbitrary point p on the projected surface appears nearer to the viewer in the stereoscopic stereopantomogram than does point m, because the former is closer to axis O_1 than is the latter and thereby also closer to the roentgen tube in passing the roentgen beam. Furthermore, all the points on the surface that are at the same height are also equidistant from the rotational axis and the viewer of the stereoscopic picture. The conical surface must therefore appear to be inclined in such a way that the apex of the cone is nearer to the viewer than is its base.

In order to check this geometric observation of his, the author

carried out the radiography of the aforementioned plastic vessel stereoscopically by using a vertical shift of the tube, the stereobase being about 5 cm. For the film he had fabricated a conical cassette of aluminium, which was somewhat larger, corresponding to the divergency rectification, than the surface of the vessel, and which was situated on the film holder at the point indicated by the "rectified" scale. The result is shown in Fig. 2. When the picture components are viewed parallelly, i.e., from the side of the roentgen tube, it is clearly seen at the center of the picture that the inclination is such that the apex of the cone is closer to the viewer than is its base. The effect thus fully corresponds to the geometric result. At the same time, however, it may be observed that the picture components do not combine properly other than in the center where the radius of each film sector lies horizontally, and, in addition, on the same straight line. This phenomenon is altogether clear and understandable. At every point the shift of the tube has taken place vertically — that is, in the direction of the rotational axes. When the picture components are subsequently straightened out into a plane, as in the present instance, they cannot combine into a stereoscopic image except for small, narrow sections at a time. If such picture components are examined stereoscopically, they must always be turned into a suitable position in relation to each other, so that the radii lie horizontally and as extensions of each other. If this is done each point separately combines well. It is further advisable to cover the other areas so they are not seen and do not "twist" the eyes. Theoretically, a concentric conical surface appears as an inclined straight plane in a stereoscopic picture, as could be demonstrated geometrically, but in practice it appears conical on account of the sector form of the picture components. If the height of the cone in relation to the diameter of the base is great, i.e., if the film is nearly vertical, a fairly broad conical surface can be seen in toto moderately well in the stereopantomogram, for the eyes — at least when accustomed to stereoscopic pictures — appear to adjust themselves rather easily to small errors in the relationship of the picture components to each other, unless the picture is viewed too long at one time. Thus, for instance, excellent stereopantomograms may be obtained of the maxilla, in which the dental arch can also be seen

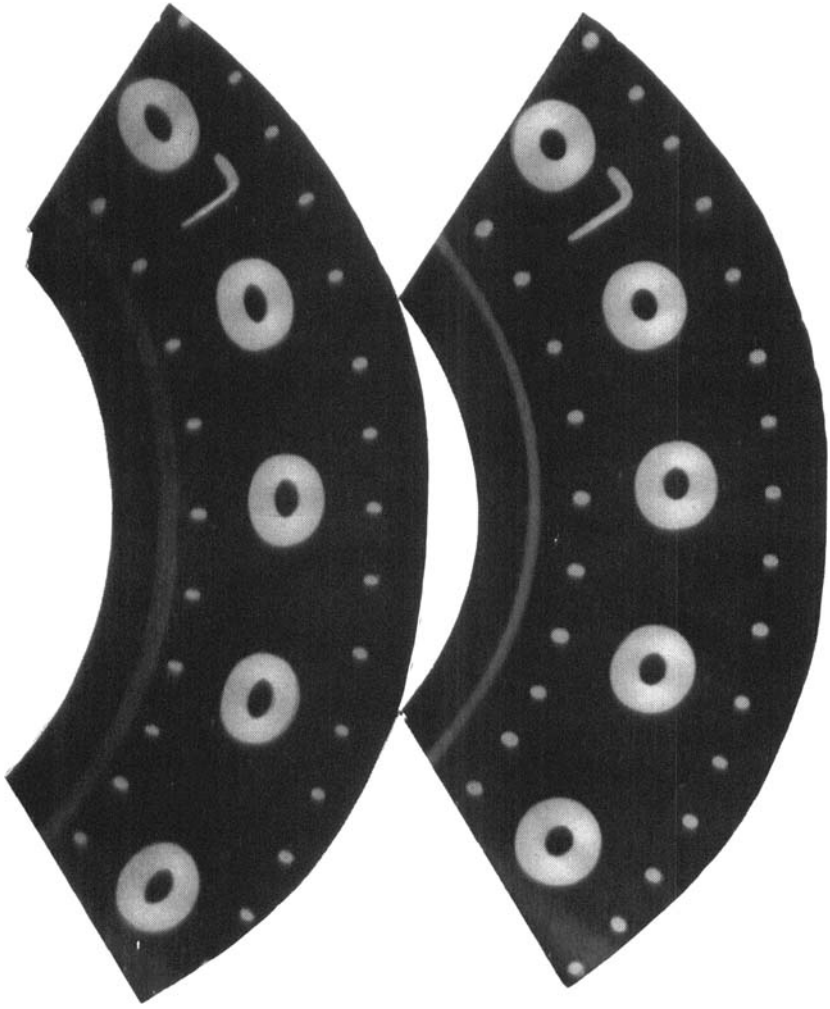

Fig. 2. Stereopantomogram of a phantom, taken as indicated in Fig. 1.

surprisingly plainly, by using slightly conical cassettes in lieu of wholly vertical ones. The maxillary teeth, as is well known, are often diverging somewhat downward, i.e., situated as if parallel to the surface of a cone. The cone formed by them does certainly not have a circular base but rather one resembling a hyperbola. If the direction of the axis of the teeth of the mandible of the patient is the same, the entire dentition is projected fairly well on the same conical film, or at least better than on an ordi-

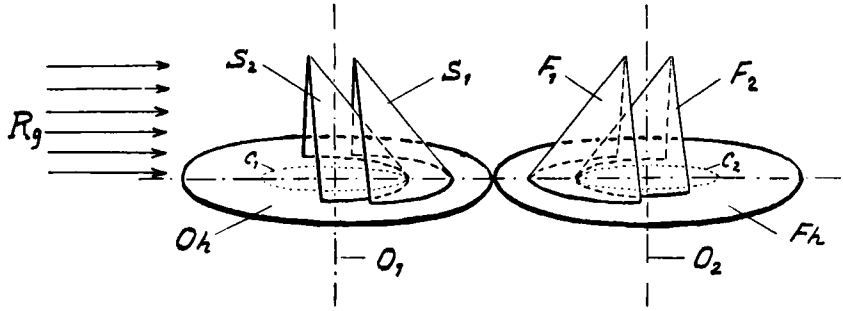

Fig. 3. Pantomographic roentgen projection of two eccentric conical surfaces S_1 and S_2 in a single exposure onto conical films F_1 and F_2.

nary vertical film. Since the conical character of the film is slight, the stereoscopic components merge readily. The conical cassette cannot lie concentrically on the film holder, for the shape of the base of the cone is not circular. Consequently, a film of the shape in question must actually be eccentric in regard to its position and represents a combination of the following two positions (Fig. 3).

In Fig. 3 two conical films have been placed on film holder Fh. The axis of film F_1 lies eccentrically in front of rotational axis O_2 and the axis of film F_2 behind it, if the side toward the roentgen tube be considered the front. The films might, of course, be exposed even by a single rotation: the image of surface S_1 would be projected onto film F_1, and the image of surface S_2 be projected on film F_2, the narrow vertical roentgen beam Rg being directed along the vertical plane on which the rotational axes O_1 and O_2 are situated. As for the shape of the conical surfaces in these eccentric cases a geometric examination shows that surface S_1 is inclined toward the viewer from the direction of the roentgen tube when one looks at the stereopantomogram parallelly. The explanation of this phenomenon is presented in conjunction with Fig. 1. Moreover, if the height of the film cone is sufficient in relation to the base to permit a simultaneous comprehension of the stereoscopic effect throughout the surface projected, the impression is one of viewing the conical shape from within, i.e., as if a concentric surface were involved. Stereopantomography of the conical plastic vessel led to corresponding results. As this stereopantomogram closely resembles Fig. 2, it will not be presented here. Furthermore, it can be demonstrated

both theoretically and experimentally that the inclination of the surface is not rectilinear anywhere else than along the midline of the surface, where the axis of the cone and the rotational axis and roentgen beam have been in the same vertical plane at a certain moment during the rotational movement. Elsewhere, the joint line of intersection of the roentgen beam and the projected surface — as well as of the film — is a hyperbola, for the axis of the cone is parallel to the rotational axis but is situated eccentrically. In this case the intersection of the hyperbola is thus behind the rotational axis (on the film) and becomes more clearly observable if the film cone is complete or at least extends farther back than the half cone in the pictures. Since the line of intersection is a hyperbola, the upper part of the surface projected is inclined more steeply toward the viewer than the lower part in the manner indicated by the hyperbola. The inclination, or, in this connection rather the bend, varies at different points, depending on how far each point is from the midline of the conical surface and how great is the eccentricity of the axis of the cone in relation to the rotational axis.

As for film F_2 and the surface S_2 projected onto it, we find both geometrically and experimentally that the matter is still more complicated (Fig. 3). If on film holder Fh we draw a concentric circle C_2 which is tangent to the circle formed by the base of film cone F_2 at the line through the center of the film, we may observe the other parts of the lower edge of the film to be outside the said circle. Likewise, the arc of conical surface S_2 is correspondingly outside circle C_1 everywhere else than at the point of tangency. Since the curve of the surface thus projected draws away from the point of tangency in each direction from the axis — and also from the viewer —, the curve must appear in the stereopantomogram to be "convex", i.e., with the center closest to the viewer. This phenomenon was perceived already in studies of the forms of vertical surfaces[8]. If the cone be cut at different heights in planes in the direction of the base, the lines of intersection of the conical surface may be observed to stand in relation to the corresponding concentric tangential circles in different ways at different heights. Considering, in addition, the fact that in this case, too, the line of intersection of the roentgen beam and the conical surface is a hyperbola, as

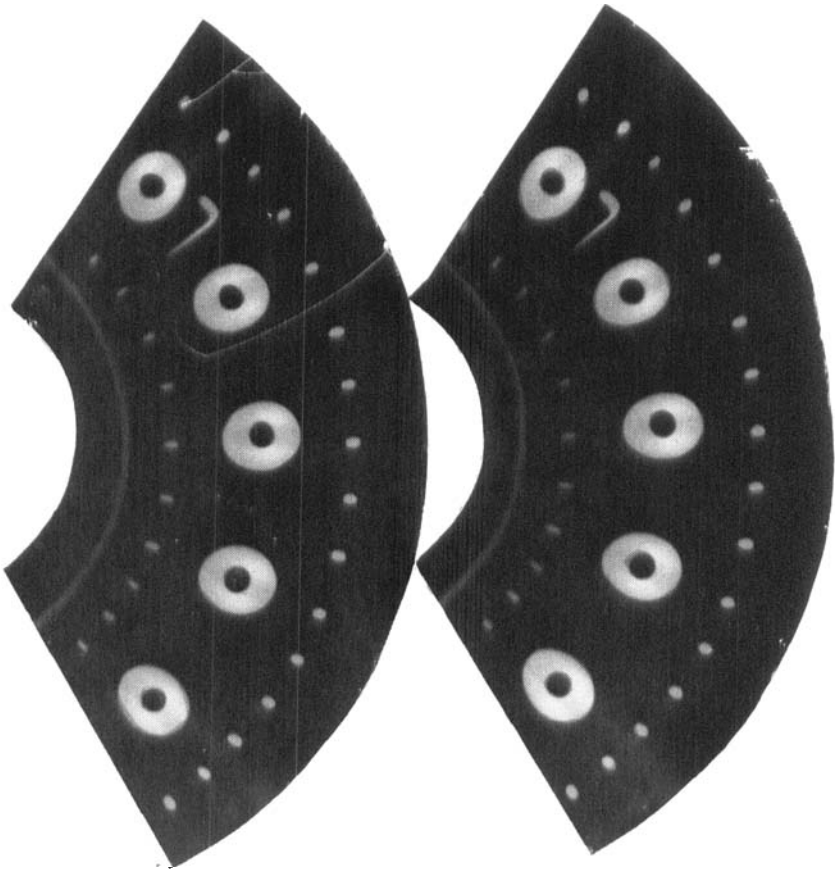

Fig. 4. Stereopantomogram of phantom corresponding to surface S_2 in Fig. 3.

in the position discussed above, it is clear that in the stereopantomogram the shape of the conical surface will be quite inexact, constituting, as it were, a kind of "compromise" between many — even opposing — factors. Thus, on the basis of the stereoscopic picture one cannot judge offhand what form of conical surface the picture depicts, unless the circumstances prevailing during the exposure are precisely known. This conception is supported by a phantom test, which the author carried out by means of the aforementioned plastic vessel with a positioning corresponding to the projected surface S_2. The result is seen in the roentgenogram in Fig. 4. In the middle part of the conical surface the original upper half of the surface, that is the part constituting the apex of the cone, is inclined somewhat

toward the viewer, but the sides — in this picture the uppermost and lowermost parts — are inclined in the opposite direction. The area lying between these parts forms a "transitional area" and in the stereopantomogram appears to be a vertical surface.

On the basis of this concise study, embracing only three chosen cases, it may be concluded that conical surfaces cannot show up well in stereopantomograms except when the inclination of the conical surface in relation to the rotational axis is very slight or the area of the surface of the projected cone is quite limited. In addition, it is evident that on the basis of a stereopantomogram it is not possible to conclude offhand what kind of surface has been projected unless the shape and position of the film and its situation in relation to the rotational axis are precisely known.

SUMMARY

From his geometric experimental studies the author concludes that conical surfaces are not suitable objects for stereo-pantomography unless the inclination of the surface, compared with the vertical plane, is very small, or unless the conical surface projected is very limited in extent horizontally.

It further appeared that the stereo-pantomogram alone does not permit conclusions as to the shape of the conical surface projected: in a stereoscopic picture the shape of a surface may be greatly distorted owing to various factors.

RESUME

SUR LES FORMES DES SURFACES CONIQUES DANS LA STEREO-PANTOMOGRAPHIE

De ses études géométriques expérimentales l'auteur tire la conclusion que les surfaces coniques ne sont pas convenables à la stéréo-pantomographie, à l'exception des surfaces dont l'angle d'inclination s'écarte très peu de l'axe vertical, ainsi que des cas où la surface conique projetée est fort limitée dans le sens horizontal.

De plus, l'étude démontre que la forme véritable d'une surface conique ne peut être déterminée directement par un stéréopantomogramme: en effet, la forme de la surface dans les images stéréoscopiques peut être très faussée, à cause de plusieurs facteurs différents.

ZUSAMMENFASSUNG

ÜBER DIE FORMEN VON KONISCHEN FLÄCHEN IN DER STEREOPANTOMOGRAPHIE

Der Verfasser ist in einer geometrisch-experimentellen Untersuchung zum Ergebnis gekommen, dass konische Flächen keinen geeigneten Gegenstand der Stereopantomographie bilden, es sei denn, dass der Neigungswinkel von der Vertikalachse nur in geringem Grad abweicht oder die aufzunehmende konische Fläche horizontal nur wenig begrenzt ist.

Aus der Untersuchung geht ferner hervor, dass die tatsächliche Form einer konischen Fläche sich nicht ohne weiteres durch ein Stereopantomogramm bestimmen lässt. Die Form der Fläche kann in einer stereoskopischen Aufnahme infolge mehrerer Faktoren in hohem Grad verzogen erscheinen.

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