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A QUANTITATIVE ANALYSIS OF THE NUMERICAL DENSITY AND THE DISTRIBUTIONAL PATTERN OF PRISMS AND AMELOBLASTS IN DENTAL ENAMEL AND TOOTH GERMS

III. THE CALCULATION OF PRISM DIAMETERS AND NUMBER OF PRISMS PER UNIT AREA IN DENTAL ENAMEL

by
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INTRODUCTION

Several contributors to dental literature have measured prism diameters in dental enamel, cf. *Köllicker* (1859), *Pickerill* (1913), *Hopewell-Smith* (1913, 1926), *Chase* (1927 a), *Bödecker* (1927), *Marcus* (1931), *Eisenberg* (1938) and *Yosida* (1938).

The results have varied from 6.8 μ (Eisenberg) to 3.4 μ (Yosida) for the outer surface, and from 2.5 μ (Pickerill) to 3.9 μ (Yosida) on the inner surface on the enamel mantle.

However, a clear definition of the conception prism diameter has not been given by any of the authors cited above, except Chase, who stated that the interprismatic substance had been included in his measurements. Therefore, it is difficult to evaluate the results presented in the literature.

Chase (1927 a) calculated the number of prisms on the outer enamel surface of each tooth in the human permanent dentition. The numbers ranged from 5 million on the second mandibular incisor to 12 million on the first maxillary molar.

Chase obviously obtained his results by dividing the measured area of the outer surface of a given tooth by πr^2, where r designates half the value of the mean prism diameter. This mean prism diameter was calculated from measurements of prism diameters on the outer enamel surface of human permanent teeth.

Fosse (1964 a) calculated a mean number of 21 904 prisms per mm^2 on the outer surface and a mean number of 47 089 prisms per mm^2 on the inner enamel surface of a bicuspid. From these means were calculated the total numbers of prisms on the outer and inner enamel surfaces of the same tooth. These mean values of the outer and inner prism densities were obtained by linear countings of longitudinally cut prisms.

No author has tried to define an interrelation between prism diameters and the number of prisms per unit area.

With the exception of *Hopewell-Smith* (1926), it seems that no author has conducted a study pertaining to the distributional pattern of prisms in cross-section. However, the forms of the cross-sectioned prisms have been described and discussed by several authors (*Chase*, 1927 b; *Smreker*, 1930; *Merkel*, 1935; *Eisenberg*, 1938; *Freiberg*, 1939; *Wolf*, 1942; *Helmcke*, 1963; *Boyde*, 1964; *Romaniuk & Schraff*, 1965; *Hinrichsen & Engel*, 1966).

Consequently, the problem was to develop a method by which one could express the mean numerical prism density and a mean defined prism diameter within a given area of cross-sectioned prisms. Likewise, it was desirable to have expressions describing any deformation of the distributional pattern of the prisms. Since the method was to be used in comparisons between different enamel regions, it would also be necessary to have expressions describing variations of density and pattern.

MATERIAL AND METHOD

A theoretical model

Let us imagine an idealized prismatic pattern where the prisms are represented by congruent tangent circles in the hexagonal distribution that yeilds the maximum number of such circles per unit area.

Such a model is depicted in Fig. 1. Parallel to and along the three directions designated by the letters V_1, V_2 and V_3 are orientated the central distances between each pair of adjacent circles. These central distances are designated by the letter d in the drawing, and each equals the diameter of the circles.

The central distances between three mutually tangent circles constitute an equilateral triangle, designated by ABC in Fig. 1. Two such triangles with one side common constitute a parallelogram where the short diagonal and the sides equal d. Such a parallelogram is drawn in Fig. 1, and its four angles are designated by the letters A, B, C', C.

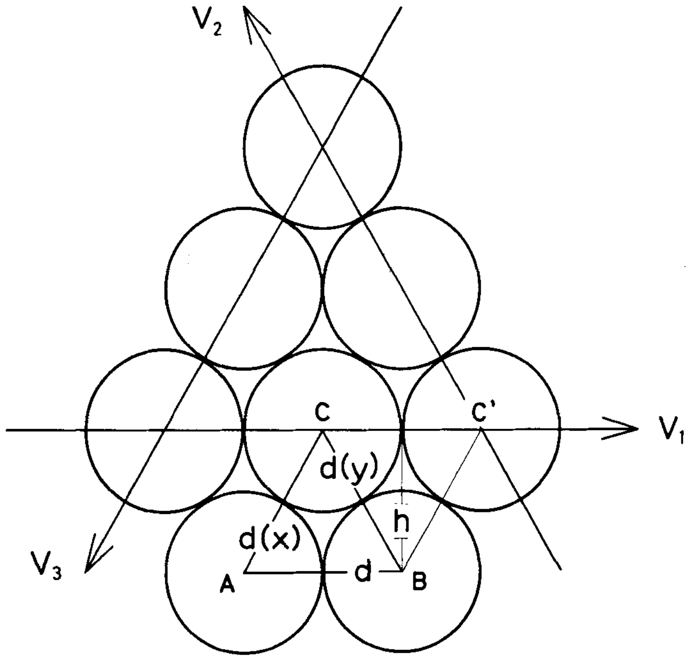

Fig. 1. A model of congruent tangent circles in symmetric hexagonal distribution. The central distances designated by the letter d, are orientated in the directions indicated by the arrows marked V_1, V_2 and V_3. The letters ABC designate the angles of a unit triangle whose sides constitute the connecting lines between the centers of three mutually tangent circles. The letters ABC'C designate the angles of a unit parallelogram which is formed by two unit triangles with one side common. The number of such parallelograms equals the number of circles per unit area.

It may be proven that the number of circles per square unit equals the number of such parallelograms.

The equation expressing the number of circles per mm^2 for the model described above is:

$$I\,a \quad \frac{2 \cdot 10^6}{d^2 \sqrt{3}}$$

when the length unit is 1 μ.

In Fig. 1 the central distance d equals the diameter of the circles. But we may also suppose that the central distance was kept constant, while the diameter was shortened. The equation above would be unaltered, since it is expressing only the number of points or centers per mm^2.

The distance between the peripheries of two adjacent circles may be considered analogous to the thickness of the interprismatic substance.

If all the central distance parallel to V_1 are designated by d, those to V_2 by y, and those to V_3 by x and if futher d, y and x are mutually unequal, the resultant model would represent points or centers in asymmetric distribution.

Even between such points a network of parallelograms may be drawn, and their number per square unit will likewise equal the number of parallelograms per square unit.

The general formula expressing the number of points distributed thus is

$$\text{II} \quad a = \frac{2 \cdot 10^6}{\sqrt{4d^2y^2 - (d^2+y^2-x^2)^2}}$$

The height on the horizontal distance d which is parallel to V_1, may be calculated thus:

$$\text{III} \quad h = \tfrac{1}{2}\sqrt{\frac{4d^2y^2 - (d^2+y^2-x^2)^2}{d^2}}$$

If we let the area of one parallelogram be expressed by d and the height h on this central distance, we get the following equation

$$\text{IV} \quad a = \frac{10^6}{d \cdot h}$$

expressing the number of points per mm².

If d, y and x are equal, then h equals the expression $\tfrac{1}{2}d\sqrt{3}$, which signnifies that $\frac{d\sqrt{3}}{2\,h} = 1$. If, on the other hand, $d > y = x$, then the ratio $\frac{d\sqrt{3}}{2\,h}$ would be greater than the value 1. And if $d < y = x$, $\frac{d\sqrt{3}}{2\,h}$ would be smaller than 1.

In triangles with sides d, y and x, this ratio may be symbolized by K:

$$\text{V} \qquad K = \frac{d\sqrt{3}}{2\,h}$$

The value of K equals the ratio between the height on the base d in an equilateral triangle and the height on the same base d in the triangle with sides d, y and x.

For a model consisting of congruent triangles with unequal sides d, y and x, the side of the equilateral traingle with the same area may be calculated by the formula

$$\text{VI} \quad D = \sqrt{\frac{2dh}{\sqrt{3}}}$$

The dimension D may be regarded as the mean central distance of the model.

Adaption of model to enamel

The theoretical considerations above concern models of congruent parallelograms.

With certain limitations, however, they prove valid for cross-sectioned prisms in the enamel.

On photographic reproductions of cross-sectioned prisms the apparent geometrical centers of the prisms were plotted. In the three main directions represented by the three arrows in Fig. 1 connecting lines were traced between each pair of adjacent centers. These connecting lines or central distances may be measured. They correspond to prism diameters including half the thickness of the interprismatic substance on either side of the prism.

The central distances that lie approximately transversal to the longitudinal axis of the tooth, are always designated by d.

Fig. 2 represents a photomicrograph from a region near the outer surface of the enamel mantle where there are but small variations in the prismatic pattern.

The plotted prism centers constitute 9 groups of fairly equal size.

The traced central distances form triangles of different shapes and areas. Each group and each triangle in each group is numbered.

Within this region the sides of each triangle are measured and their actual sizes are calculated by means of a microscale photographed, using the same magnification. All central distances transversely orientated to the long axis of the tooth are designated by d, all parallel to the V_2 direction by y and all parallel to the V_3 direction by x.

The simplest way of obtaining an expression of the number of prisms per mm^2 is to calculate the three means $\langle d \rangle$, $\langle y \rangle$ and $\langle x \rangle$ from all the measured distances within the region. These means are then inserted into formula II. The number of prisms per mm^2 calculated in this way will be symbolized by a_{dyx}. The ratio K and the mean central distance D may also be calculated from $\langle d \rangle$, $\langle y \rangle$ and $\langle x \rangle$.

The accuracy of the resultant a_{dyx} is dependent, however, on the degree of variation in prism size within the pattern. In addition, it is dependent on the distribution and the number of plotted prisms within the region.

A more correct expression of the number of prisms per mm^2 is obtained by calculating the product $d \cdot h$ for each triangle and inserting the mean $\langle dh \rangle$ into formula IV. The resultant a-value will be symbolized by a_{reg}.

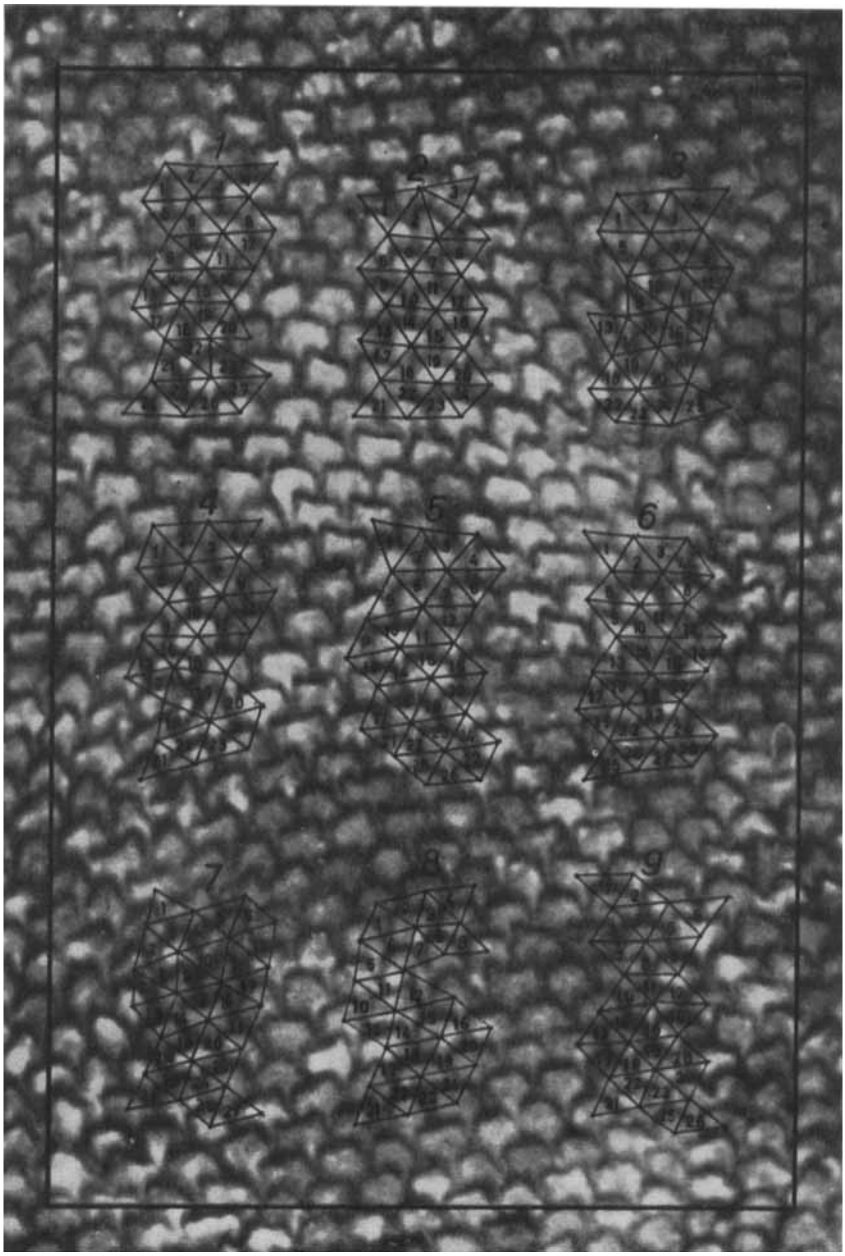

Fig. 2. Photomicrograph from the outer enamel surface. This region is called T_1 in the text. The area of the great rectangle is 15,000 μ^2. 452 prism centers were counted within the rectangle. In 9 groups the central distances between adjacent prisms have been traced forming continuous networks of unit triangles. The groups and triangles are numbered. Each central distance was measured. The greater side of the rectangle is parallel to the long axis of the tooth.

$d \cdot h$ represents the area of a parallelogram. Within the region, however $\langle dh \rangle$ is equal to the theoretical mean area of the cross-sectioned prism and half the thickness of the surrounding interprismatic substance.

To express the variation in prism density, i.e. the number of prisms per unit area within one region, the a-value may be calculated from the $\langle dh \rangle$ value for each group. From these group values which are designated by a_{gr}, is calculated the mean $\langle a_{gr} \rangle$ and the standard deviation $S_{a_{gr}}$. The accuracy of $\langle a_{gr} \rangle$ is dependent on the distribution and the number of plotted prisms within the region. The accuracy of a_{gr} is not dependent on the variation in prism size within the group.

The ratio $\langle K \rangle$ is the mean of the K-values of all the triangles within the region. Thus a standard deviation is expressible and is designated by S_K.

When $\langle K \rangle$ approximately equals the value 1, and when $\langle y \rangle \approx \langle x \rangle$, a tendency towards a symmetric hexagonal distribution of the prisms within the region is indicated.

When $\langle K \rangle$ is markedly greater than the value 1, and when $\langle y \rangle \approx \langle x \rangle$, a compression of the prism pattern in a direction parallel to the long axis of the tooth is indicated (*Fosse*, 1964 b).

The ratio K does not describe the ratio between y and x. Measurements conducted in numerous regions in different human teeth seem to indicate, however, that $\langle y \rangle$ and $\langle x \rangle$ for practical purposes are equal, and that $\langle d \rangle$ is either practically equal to or greater than $\langle y \rangle$ or $\langle x \rangle$. Usually, therefore, $\langle K \rangle$ alone is a sufficient description of the prismatic pattern without reference to $\langle d \rangle$, $\langle y \rangle$ and $\langle x \rangle$.

For each triangle with unequal sides, the side D of the equilateral triangle with the same area is calculated. The symbol $\langle D \rangle$ then designates the mean of the D-values of all the triangles within the region. $\langle D \rangle$ may be interpreted as the mean size of the prism diameter within the region, including the interprismatic substance, and may thus be comparable to the length of the prism diameter given by other authors. A standard deviation is expressible and is symbolized by S_D.

The selection of three different enamel regions for the calculation of a-, K- and D-values

Three test regions were photographed and copied on 100 % unschrinkable paper. The first, which is represented by Fig. 2, is later referred to as T_1. The next two regions which are represented by Figs. 3 and 4, are later referred to as T_2 and T_3 respectively.

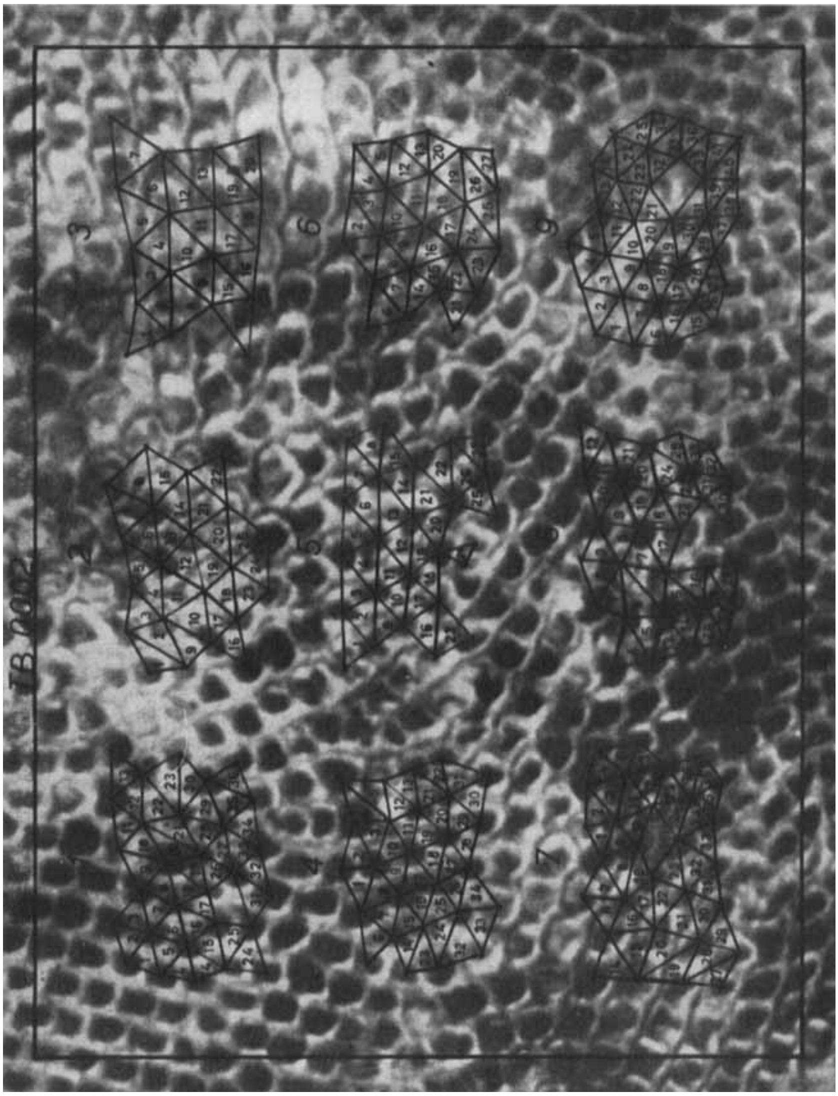

Fig. 3. Photomicrograph from the enamel near the amelodentinal junction. This region is called T_2. The area of the great rectangle is 17.357 μ^2. 467 prism centers were counted within the rectangle. The central distances between adjacent prisms have been traced in 9 groups. The shorter side of the rectangle is parallel to the long axis of the tooth.

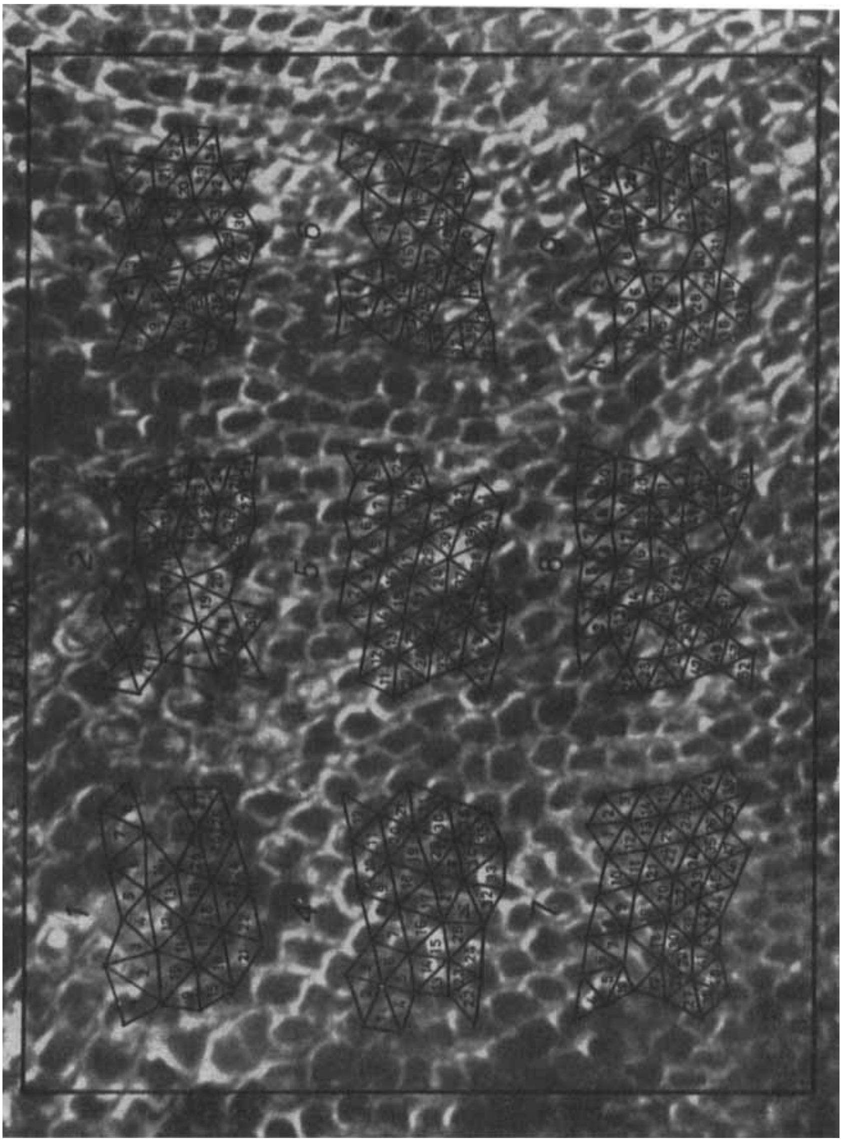

Fig. 4. Photomicrograph from the enamel near the amelodentical junction. This region is called T_3. The area of the great rectangle is 17,357 μ^2. 561 prism centers were counted within the rectange. The central distances between adjacent prisms have been traced in 9 groups. The shorter side of the rectangle is parallel to the long axis of the tooth.

T_1 is a region near the outer enamel surface, while T_2 and T_3 are regions near the amelodentinal junction where generally the prismatic pattern seems to be most irregular in the enamel of human permanent teeth.

T_2 and T_3 represent two regions selected from an extensive material for their great irregularity of prism size and pattern combined with distinctness of prism contours. Regions of this size usually include areas in which the prisms seem to be confluent and where it is difficult to determine the exact number.

On each of the three copies represented by Figs. 2, 3 and 4, a great rectangle has been drawn, the actual area of which has been calculated. This area will be designated by F.

All the prism centers within the rectangle were counted. All centers lying on two of the rectangle sides were included, while all centers on the other two were excluded from the final sum.

The actual number within F was multiplied by $\frac{1mm^2}{F}$. The theoretical resultant number of prisms per mm^2 is called a_t. For each teast region the calculations described in the former chapter were performed.

RESULTS

Application and testing of method

Elimination of triangles

From each of the 9 groups within T_1, T_2 and T_3, about 7 triangles were selected, giving approximately 63 per test region. The selected triangles were situated along the margins of each group. For this selection the same calculations were carried out as for the full number of triangles.

From this first selection 3 triangles were further picked from each group, making 27 triangles in all, representing the region. The same calculations were carried out for the second selection.

The results of all these calculations for T_1, T_2 and T_3 are listed in Tables I, II and III.

Tables IV, V and VI present the values of a_{gr} and $\langle K \rangle_{gr}$ for each group, calculated for all the triangles and for the two selections. For each group the number of triangles is listed at the left-hand side. These tables demonstrate that a_{gr} varies less in T_1 than in T_2 and T_3.

The maximum deviation from a_t was found in T_3 where $\langle a_{gr} \rangle$ for the first selection of 68 triangles equalled 33 995 prisms while a_t equalled 32 321 prisms. The difference constituted 5.1 % of a_t.

Table I

Prism density and pattern near the outer enamel surface. Test region T_1 Fig. 2. Elimination of triangles

Tri-angles	Groups	Number of prisms/mm²				Vert. compr.		Centr. dist. in microns	
		a_{dyx}	a_{reg}	$\langle a_{gr} \rangle$	S_{agr}	$\langle K \rangle$	S_K	$\langle D \rangle$	S_D
232	9	30083	30421	30452	1379	1.31	0.16	6.15	0.36
62	9	29847	30105	30175	1225	1.30	0.14	6.18	0.30
27	9	29966	30066	30224	2110	1.30	0.15	6.19	0.42

The rectangle contains 452 prism centers.
F = 15,000 μ^2): a_t = 30133 prisms/mm²

Table II

Prism density and pattern near the amelondentinal junction. Test region T_2 Fig. 3. Elimination of triangles

Tri-angles	Groups	Number of prisms/mm²				Vert. compr.		Centr. dist. in microns	
		a_{dyx}	a_{reg}	$\langle a_{gr} \rangle$	S_{agr}	$\langle K \rangle$	S_K	$\langle D \rangle$	S_D
279	9	26168	26787	26635	4132	1.19	0.22	6.52	0.75
68	9	25928	26093	26869	3909	1.22	0.18	6.62	0.61
27	9	26209	26852	27580	4545	1.24	0.20	6.55	0.65

The rectangle contains 467 prism centers.
F = 17,357 μ^2): a_t = 26905 prisms/mm²

Table III

Prism density and pattern near the amelodentinal junction. Test region T_3 Fig. 4. Elimination of triangles

Tri-angles	Groups	Number of prisms/mm²				Vert. compr.		Centr. dist. in microns	
		a_{dyx}	a_{reg}	$\langle a_{gr} \rangle$	S_{agr}	$\langle K \rangle$	S_K	$\langle D \rangle$	S_D
352	9	32053	32622	32454	4486	1.07	0,20	5.91	0.61
68	9	32189	32727	33995	5707	1.07	0,21	5.89	0.74
27	9	31940	32102	33333	6018	1.06	0,25	5.99	0.72

The rectangle contains 561 prism centers.
F = 17,357 μ^2): a_t = 32321 prisms/mm²

The maximum deviation of a_{reg}, representing all triangles, was found in T_3, where a_{reg} equalled 32 622 prisms while a_t equalled 32 321 prisms. The difference constituted 0.93 % of a_t.

Table IV

Prism density and pattern in the 9 groups of test region T_1 Fig. 2. Elimination of triangles

△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$
27	30179	1.31	24	29891	1.30	25	29802	1.17
7	31034	1.35	7	28113	1.20	7	30439	1.18
3	30988	1.36	3	26631	1.30	3	32829	1.09
24	28173	1.27	27	28694	1.33	28	32054	1.41
7	29233	1.33	8	29638	1.31	7	32707	1.38
3	28538	1.30	3	28216	1.32	3	32488	1.39
27	32016	1.25	24	31290	1.33	26	31971	1.44
6	30533	1.22	7	29376	1.32	6	30499	1.44
3	32488	1.21	3	28843	1.32	3	30998	1.41

		a_{reg}	$\langle K \rangle$	
All groups	232	30421	1.31	a_t = 30133
	62	30105	1.30	
	27	30066	1.30	

Table V

Prism density and pattern in the 9 groups of test region T_2 Fig. 3. Elimination of triangles

△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$
36	29253	1.19	25	22547	1.32	20	17860	1.14
7	31515	1.11	8	23970	1.39	8	19519	1.21
3	31075	1.07	3	24503	1.36	3	19817	1.22
34	30473	1.24	27	24695	1.36	27	26029	1.12
7	33574	1.11	8	24514	1.45	8	25842	1.10
3	37216	1.14	3	25425	1.52	3	26413	1.12
36	28132	1.15	33	28132	1.15	41	31815	1.10
8	27948	1.17	7	27666	1.27	7	27276	1.17
3	29069	1.12	3	28481	1.36	3	26226	1.23

		a_{reg}	$\langle K \rangle$	
All groups	279	26787	1.19	a_t = 26905
	68	26093	1.22	
	27	26852	1.24	

Table VI

Prism density and pattern in the 9 groups of test region T_3 Fig. 4. Elimination of triangles

△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$	△	a_{gr}	$\langle K \rangle_{gr}$
27	23992	1.28	30	27392	0.99	36	34006	1.04
8	21146	1.25	8	34807	1.02	7	31951	1.07
3	21630	1.23	3	30220	0.92	3	29498	1.04
36	29036	1.13	45	33249	1.03	40	36888	1.01
7	29762	1.13	7	35464	1.06	8	40777	1,03
3	28449	1.30	3	33840	1.03	3	40584	0.96
47	36900	1.12	53	37902	1.02	38	32719	1.04
7	41299	1.17	8	35473	0.96	8	35276	0.98
3	40338	1.29	3	37878	0.82	3	37565	0.97
				a_{reg}	$\langle K \rangle$			
All groups			352	32622	1.07	a_t = 32321		
			68	32727	1.07			
			27	32102	1.06			

Elimination of groups

According to the tables, the greatest variation of a_{gr} was found in T_3.

For this region, therefore, a_{reg} and $\langle K \rangle$ were calculated after elimination of groups in different combinations. The results are presented in Table VII.

In the left-hand column the numbers of the groups included in each experiment are given.

These six experiments yielded 33 068 prisms per mm² as the maximum a_{reg}, and 32 331 prisms per mm² as the minimum a_{reg}. The difference between the maximum value and at represents 2.33 % of a_t.

Table VII

Prism density and pattern in test region T_3 Fig. 4. Elimination of groups

Experiment	Groups included	a_{reg}	$\langle K \rangle$
1	1, 3, 5, 7, 9	32331	1.09
2	2, 4, 5, 6, 8	33068	1.07
3	1, 2, 3, 5, 7, 8, 9	32605	1.07
4	1, 3, 4, 5, 6, 7, 9	32435	1.08
5	2, 3, 4, 5, 8, 9	32583	1.04
6	1, 2, 5, 6, 7, 8	33025	1.06
	All groups	32622	1.07

The maximum difference by the calculation of a_{reg} in this experiment would occur if only one of the groups represented the region, and if this group consisted of the greatest or the smallest prisms. Groups number 1 and number 8 represented such extreme values for T_3, since $a_{grl} = 23\,992$ and $a_{gr}1$, $a_{gr}8 = 37\,902$.

The ratio ⟨K⟩ was found to be 1.07 when all the groups and triangles were included. By the elimination of the groups a maximum ratio of 1.09 and a minimum ratio of 1.04 occured.

Testing of ⟨K⟩

The value of a_{reg} was compared to a_t. The ratio ⟨K⟩ may likewise be roughly tested by measuring the distances between the first and the last of a given number of prisms in linear rows in the three directions V_1, V_2 and V_3 within the region. This was done for 5 rows in each of the three directions in T_1. The mean length in the V_1 direction was 9.0 cms, in the V_2 direction 7.4 cms and in the V_3 direction 7.4 cms. The relationship between these three lenths should correspond to the relationship between ⟨d⟩, ⟨y⟩ and ⟨x⟩. The ratio K was calculated for the lengths and gave the value 1.32. In T_1 ⟨K⟩ equalled 1.31.

Plotting and measuring errors

The variation within the prismatic pattern expressed by S_{agr}, S_K and S_D is due partly to measuring errors and partly to variation in the subjective interpretation of the prism center. The latter error is henceforth called the plotting error.

Plotting errors in the middle of a prism group will not affect the resultant a_{gr}; neither will measuring errors if they are not systematic. The errors will neutralize one another. Plotting errors at the borders of the group, however, will affect the result. Such marginal errors are included in the resultant a_{gr} and $\langle K\rangle_{gr}$ and thus also in a_{reg} and ⟨K⟩.

To determine the magnitude of the marginal plotting error and the measuring error, one group of 16 prisms of fairly variable form and size was plotted 9 times, with a time lapse of a full year between the first and the last plotting.

In Table VIII are given the results for a_{gr}, $\langle K\rangle_{gr}$ and ⟨D⟩gr. The standard deviations for the 9 values for a_{gr}, $\langle K\rangle_{gr}$ and $\langle D\rangle_{gr}$ have also been presented in the table.

$S_{agr} = 383$ may be regarded as a general estimate of the plotting and measuring error combined and may thus be compared with S_{agr} for any region.

Table VIII

Marginal plotting and measuring errors by 9 plottings of the same prism group consisting of 18 triangles and 16 prisms

Plott. no.	Prisms/mm²	Vert. compr.		Centr. dist. in microns	
	agr	⟨K⟩gr	SK	⟨D⟩gr	SD
1	32408	1.15	0.19	5.95	0.35
2	33210	1.13	0.15	5.88	0.28
3	32083	1.17	0.14	5.99	0.31
4	32266	1.18	0.13	5.97	0.25
5	32617	1.14	0.16	5.94	0.28
6	33166	1.16	0.18	5.89	0.29
7	32652	1.16	0.16	5.93	0.31
8	32336	1.14	0.18	5.97	0.23
9	32988	1.15	0.13	5.90	0.33
Mean	32636	1.15		5.94	
St.dev.	383	0.015		0.036	

Table IX

Plotting and measuring errors in K and D by 9 plottings of 4 different triangles

Plott. no.	Vert. compression				Central distance in microns			
	K_1	K_2	K_3	K_4	D_1	D_2	D_3	D_4
1	1.54	1.54	1.25	1.41	5.75	5.75	5.43	6.01
2	1.34	1.46	1.10	1.36	5.86	5,91	5.45	5.82
3	1.39	1.39	1.09	1.23	6.06	6.06	5.81	6.11
4	1.34	1.51	1.25	1.25	5.86	6.10	5.75	5.75
5	1.42	1.47	1.04	1.05	5.69	5.88	5.95	6.27
6	1.60	1.47	1.18	1.18	5.37	5.88	5.91	5.91
7	1.34	1.59	1.27	1.32	5.86	5.94	6.01	6.22
8	1.46	1.46	1.30	1.30	5.91	5.91	5.63	5.94
9	1.38	1.27	1.04	1.23	6.01	6.01	5.59	6.11
Mean	1.42	1.46	1.16	1.25	5.82	5.93	5.72	6.01
St. dev.	0.088	0.086	0.097	0.100	0.19	0.10	0.20	0.16

$S\langle K\rangle gr = 0.015$ and $S\langle D\rangle gr = 0.036$ also represent measuring plus marginal plotting errors. But these values cannot be compared with S_K and S_D since the latter include not only marginal plotting errors but also plotting errors in the middle of the groups.

To evaluate the inherent errors of S_K and S_D the 9 K and D-values for the first four triangles were tabulated and their standard deviations were calculated for all four triangles separately. The results are presented in Table IX.

The standard deviations at the bottom of the table express measuring and plotting errors and may be compared with S_K and S_D for any group or region.

The inherent plotting and measuring errors of the actually measured central distances were calculated in a similar manner, using the first 4 triangles of the 9 plottings. The result concerning d was approximately 0.25 μ.

DISCUSSION

Chase (1927) tried to determine the mean area of the cross-sectioned prisms. The total outer surface of the enamel mantle was divided by this area to determine the approximate total number of prisms on the outer surface. Chase did not explicitly state how this area was calculated.

Fosse (1964 a) calculated an average prism number per mm^2 by squaring the number of prisms counted per length unit and offered no expression comparable to the conception prism area or prism diameter.

By the present method the number of prisms per mm^2 is calculated by dividing 1 mm^2 by ⟨dh⟩. The value of ⟨dh⟩ must be regarded as the theoretical size of the mean prism area plus half the thickness of the surrounding interprismatic substance.

The direct counting of cross-sectioned prisms within areas of known size is the simplest method of assessing the number of prisms per square unit.

For the three test regions the prisms within the rectangles were counted and from the results were calculated the number of prisms per mm^2. However, a_t also included marginal errors which originated while counting the centers along the borders.

It is difficult to find an area of the size of 15,000 μ^2 or above, where all the prisms have been made visible by the applied staining method. If, therefore, the number of prisms per square unit should be assessed by counting, such counting would have to be carried out in several smaller areas within the greater region. The border error would thus be greater than by counting all the prisms within the region. One would also have to use counting areas of varying form and size.

Also, simply counting the prisms would not give information about prism-distribution, ⟨K⟩, and the variations in prism size and diameter.

In this paper the central distances have been identified with prism diameters if the latter include the interprismatic substance. This definition was also given by Chase (1927 a).

Such prism diameters are, however, only identical to the central distances if they are orientated in the directions V_1, V_2 and V_3. These directions are determined by the distributional pattern of the prism centers.

Within a given distributional pattern of prism centers any prism form and combination of forms may be imagined; elipses or hexagons, open arcades or irregular polygons, and all with varying amounts of interprismatic substance. Therefore, neither the diameters of prisms in arbitrary directions, nor their form, determine their number or distributional pattern.

In this paper the number of prisms per unit area is always related to the unit 1 mm^2. This does not mean that a_{reg} represents a sample from 1 mm^2 surrounding the plotted region, since this constitutes only about 1/60 of 1 mm^2 for the three test regions presented. 1 mm^2 is used as the most perspicuous unit. The a-values, therefore, always refer to the plotted region, and represent only the actually plotted prisms within the region.

For each test region there was good agreement among the three values of a_{reg} representing all the triangles and the two selections of triangles. The tables IV, V and VI demonstrate directly that the greatest difference between a_t and a_{reg} would occur if too few groups represented the region.

The varying values of a_{reg} and ⟨K⟩ in table VII also demonstrate how these values would vary if different plottings of the same region were carried out, and if the number and distribution of the representative groups had been chosen differently in each plotting. This test then evaluates the deviations occuring when determining the number of prisms per mm^2 by the method presented in this paper.

It is immediately evident that the maximum difference between a_{reg} and the true density *might* occur in any region if it were represented by only one triangle. Since a_{reg} is the correct representation of the density of the plotted prisms, it is obvious that the number of prisms within any region may be calculated as accurately as desired by increasing the number of plotted prisms. When all prisms within the region are plotted, the best representation of the true density of the region has been obtained. However, the marginal error is inevitable.

It is also abvious that a good approximation to the true density would be obtained if an adequate distribution of a few triangles within the region could be chosen. An adequate distribution would mean a »balanced» representation of prism sizes about the »mean prism size». It is of course not possible to carry out a »balanced» plotting. However, by covering the region evenly the resultant a_{reg} should always be a far better estimate of the true density than a_{reg} calculated from the smallest or the biggest triangle alone. Therefore, an even distribution of plotted groups within the region is aimed at, rather than a higher number of prisms within few groups in an uneven distribution.

By the plotting of one group 9 times an expression was obtained describing the measuring error and the marginal plotting error affecting the resultant a_{gr}, see Table VIII.

The magnitude of these errors indicated that S_{agr} genarally describes a genuine variation in prism size.

S_{agr} is, however, dependent on the number, size and location of the groups within the region and must not be taken as the absolute definition of the variations in prism size. Comparisons between different S_{agr} values representing different regions seem safe, however.

Table IX demonstrated for one test group that a considerable percentage of S_K and S_D consisted of measuring and plotting errors. The absolute values of these expressions are therefore not representative of the real variations in prism pattern. In comparisons between groups and regions they may be useful, however.

The variation of actually measured central distances within a region might be expressed by the standard deviations of d, y and x and symbolized by S_d, S_y and S_x. They are always greater than S_D. The plotting and measuring errors in these standard deviations have been calculated and average 0.25 micron. This value approximates the size of the difference between S_d, S_y or S_x and S_D, which averages 0.39 micron and is quite constant for all S_{agr} and S_D. Therefore S_D may be a good estimate of the real variation of the prism diameters, although by definition S_D reflects the variation of $d \cdot h$, cf. formula IV. The method described above was used in investigations to be presented in a series of papers in this journal.

SUMMARY

The author introduced the concept central distance between the apparent centers of two adjacent prisms. Within groups of cross-sectioned prisms the apparent center of each prism was plotted. The groups were evenly distributed within a region of a given extent.

Within each group every central distance between each pair of adjacent prisms was measured.

The results were treated mathematically to obtain expressions describing the number of prisms, their distributional pattern and the variations in their dimensions within the region.

The central didtances might also be regarded as the diameters of the prisms under certain reservations.

The author demonstrated how he had tested the method by counting all the prisms within three different test regions of known area. Within the same regions the numbers of prisms had been calculated by the method outlined above.

The results were listed in tables. In the tables a_t designated the number of prisms per mm², calculated by direct counting, a_{reg} designated the number of prisms per mm² calculated by means of the central distances, and ⟨D⟩ designated the mean central distance of the prisms. The results may be condensed as follows:

Test region	a_t	a_{reg}	⟨D⟩
1	30 133	30 421	6.15 microns
2	26 905	26 787	6.52 »
3	32 321	32 622	5.91 »

The tests also demonstrated that plotting a few prisms, if they were uniformly distributed within the region, yielded a good approximation to the counted number within the same region.

The method makes it possible to detect and describe any compression or distension of the prismatic pattern directed in the plane where the measurements are carried out.

RÉSUMÉ

CALCUL DU DIAMETRE DES PRISMES ET DU NOMBRE DE PRISMES PAR UNITÉ DE SURFACE DANS L'ÉMAIL DENTAIRE

La notion de distance centrale entre les centres apparents de deux prismes adjacents est introduite par l'auteur. Dans des groupes de prismes en coupe transversale, le centre apparent de chaque prisme a été marqué. Les groupes étaient répartis uniformément dans une région d'étendue déterminée.

Dans chaque groupe, on a mesuré les distances centrales entre chaque paire de prismes adjacents.

Par traitement mathématique des résultats, on a obtenu des expressions descriptives du nombre de prismes, de leur mode de répartition et des variations de leurs dimensions dans la région.

A certaines réserves près, les distances centrales pouvaient aussi être considérées comme les diamètres des prismes.

L'auteur a montré comment il a fait l'épreuve de la méthode en comptant tous les prismes dans trois régions différentes de surface connue. Dans les mêmes régions, le nombre de prismes avait été calculé par la méthode décrite ci-dessus.

Les résultats ont été notés sous forme de tableaux dans lesquels a_t désigne le nombre de prismes par mm², calculé en comptant directement, a_{reg} désigne le nombre de prismes par mm², calculé au moyen des distances centrales des prismes, et ⟨D⟩ désigne la moyenne des distances centrales des prismes. Les résultats peuvent être résumés comme suit:

Testregion	a_t	a_{reg}	⟨D⟩
1	30 133	30 421	6.15 microns
2	26 905	26 787	6.52 »
3	32 321	32 622	5.91 »

Les tests ont aussi mis en évidence que, s'ils sont uniformément répartis dans la région, le tracé de quelques prismes donne une bonne approximation du nombre trouvé en comptant pour la même région.

Cette méthode permet de déceler et de décrire toute compression ou dispersion de la distribution des prismes dans le plan où les mesures sont pratiquées.

ZUSAMMENFASSUNG

DIE BERECHNUNG VON PRISMENDURCHMESSER UND ANZAHL DER PRISMEN PER FLÄCHENEINHEIT IM ZAHNSCHMELZ

Der Verfasser introduzierte den Begriff Zentralabstand zwischen den scheinbaren Zentren von zwei beiliegenden Prismen. Innerhalb Gruppen von quergeschnittenen Prismen wurde das scheinbare Zentrum jedes Prismas gemarkt. Die Gruppen waren gleichmässig verteilt innerhalb einem Bereiche einer bestimmten Grösse.

Innerhalb jeder Gruppe wurde der Zentralabstand zwischen jedem Paar von benachbarten Prismen gemessen. Alle Zentralabstände wurden danach matematisch behandelt um Ausdrücke zu erreichen, die Anzahl, Muster und deren Variationen beschrieben.

Die Zentralabstände möchten auch als Prismendurchmesser betrachtet werden.

Es wurde demonstriert wie die Methode kontrolliert geworden war, durch die Zählung von allen Prismen innerhalb drei verschiedener Testregionen von bekannter Grösse.

Innerhalb derselbigen Regionen waren auch die Anzahlen mittels der oben beschriebenen matematischen Methode berechnet worden. Die Ergebnisse wurden tabellarisch dargestellt. In den Tabellen bezeichnete a_t die Zahl der Prismen per mm^2, die durch unittelbare Zählung berechnet war, weil a_{reg} bezeichnete die Zahl per mm^2, die mittels der Zentralabstände berechnet war. $\langle D \rangle$ bezeichnete den mittleren Zentralabstand der Prismen in der Region.

Die Ergebnisse möchten folgendermassen zusammengefasst werden:

Testregion	a_t	a_{reg}	$\langle D \rangle$
1	30 133	30 421	6.15 microns
2	26 905	26 787	6.52 »
3	32 321	32 622	5.91 »

Die Versuche demonstrierten auch dass man nur wenige Zentralabstände innerhalb einer Region messen brauche, um ein ganz genaues Resultat zu erreichen.

Die Methode ermöglicht das Nachweisen und quantitative Beschreibung aller systematischen Deformationen des Musters der quergeschnittenen Prismen.

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