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CHANGES IN VOLUME OF THE GINGIVAL TISSUE
A STEREOPHOTOGRAMMETRIC STUDY

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INTRODUCTION

Swelling of gingival tissue is a rather common occurrence, usually of inflammatory origin. In order to study the state of the gingiva an accurate method of measurement would be of great value, as this would permit an evaluation of the extent of the pathological process. It would also offer a possibility to study the course of the pathological process over a certain period of time, for instance the time taken for regeneration of the tissue after treatment. Evidently an objective method of measurement must be applied which can reproduce geometrical data of well defined quality. If a photographic technique is used for registering tissue changes it must be based on photogrammetric principles. Photogrammetry, the science of measurements on photographs, is used for determining geometrical data such as size, position and shape of photographed objects (cf. *Hallert* 1960, 1964).

Attention has not always been paid to fundamental photogrammetric requirements in odontological photography, the reason probably being the necessity to develop qualified techniques for determination of the relations between projection centre, object and film (inner and outer orientation). In many problems of medicine and odontology, however, photogrammetry is the only means available for obtaining objective measurements. A photogrammetric method has been used in the field of medicine by *Lacman* (1950), *Adams-Ray & Hjelmström* (1951) and others. For odontological radiographic purposes a method that allows objective measurements on radiographs was designed by *Berghagen* (1951). *Björn et. al.* (1954) made a study of the swelling of the face after surgery by means of a stereophotogrammetric method. The patient was fixed to the camera system in order to facilitate the orientation. *Forsslund* (1959) presented a stereophotogrammetric method with which he studied the subepithelial blood vessels of the gingiva.

Measurements of changes in volume of the gingiva have been made by *Holm et al.* (1967) using a photogrammetric method introduced by *Holm & Krakau* (1965) and originally intended for measuring the volume of small superficial tumours. In their study the relative volume of the gingival tissue was determined by means of a grid technique. Parallel lines were projected on the teeth and the gingival margin to be examined and subsequently photographed. The volume of the marginal gingiva was then calculated by measuring a number of grid areas.

The aim of the present investigation was to develop a method for determination of gingival volume changes and to analyse its performance.

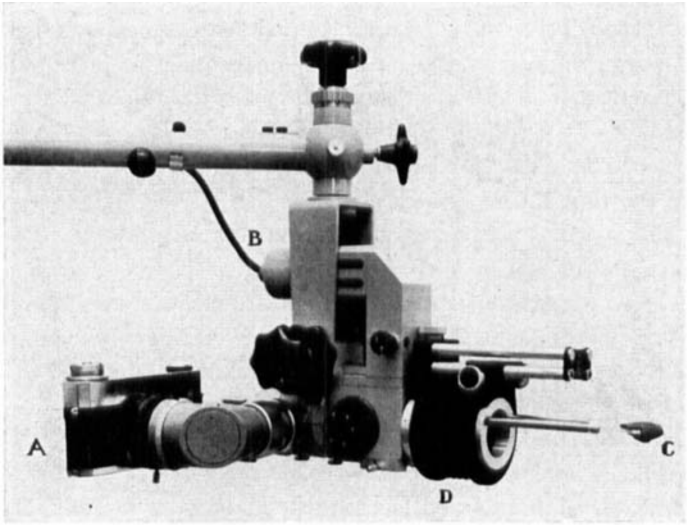

Fig. 1 a.

Zeiss stereomicroscope
A = camera housing
B = light source
C = biting plate
D = focusing device

METHOD

A. Apparatus

A Zeiss stereomicroscope fitted with two cameras with picture format 24× 36 mm was used for stereoscopic study of the object and for objective three-dimensional measurement. The negative scales used were 1:0.45, 1:0.7, 1:1.1, 1:1.8, and 1:2.75 (Fig. 1 a, b).

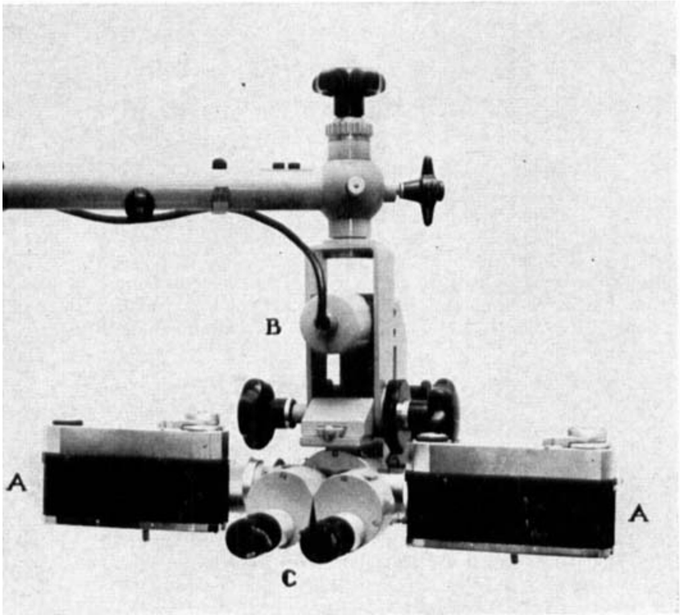

Fig. 1 b.
Zeiss stereomicroscope
A = camera housing
B = light source
C = oculars

The cameras were equipped for simultaneous exposure and the object could be studied through the oculars at the moment of exposure (Fig. 2). In order to secure the inner orientation, all movable parts were sealed.

B. Photography

By means of impression paste a biting plate was fixed in the mouth (Fig. 3). This plate was attached to the microscope in such a way that it could be re-

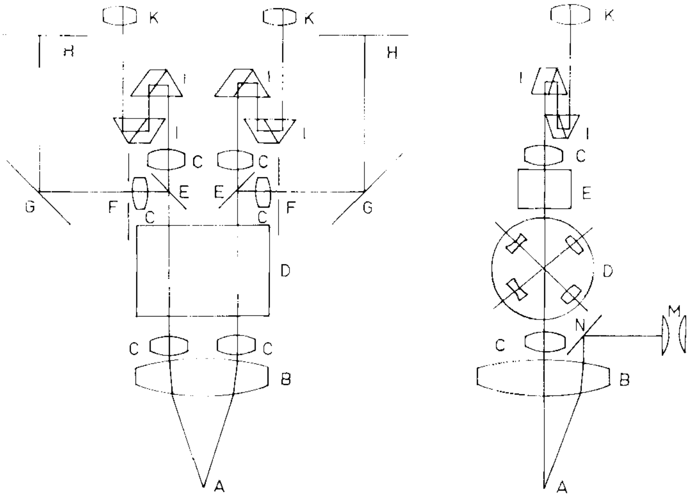

Fig. 2. Diagram of lens system of the stereomicroscope A = object, B = objective, C = lenses of the optical system, D = drum for changing magnification, E = semitransparent mirror, F = shutter of camera, G = mirror, H = negative, I = prisms, K = ocular, L = lamp, M = condensing lens, N = mirror for illumination of object.

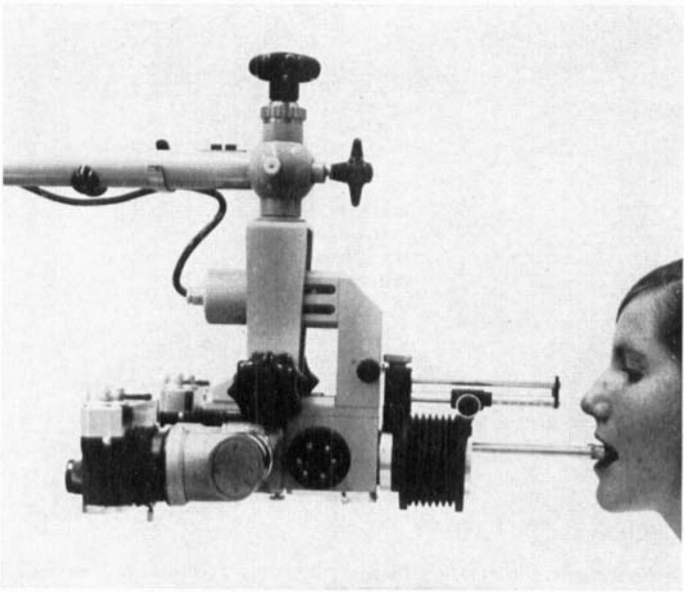

Fig. 3. Patient attached to stereomicroscope

moved and re-attached in approximately the same position. Differences in orientation were corrected from measurements made.

The biting plate made it possible to have approximately the same outer orientation for photographs taken at different times and also reduced the risk of lack of sharpness due to movement.

C. Evaluation

From the negatives, diapositives were made with such geometrical data and on such a scale that an optical model of the photographed object could be formed in a Wild A7 stereoautograph. The procedure will later be described in detail. Briefly, however, the object is mapped in a reference system defined by details on the teeth. By means of stereomodels obtained from photographs taken at different times, changes in swelling can be determined. The shape of the gingiva may be shown graphically by means of a number of sections spaced one millimetre apart. By superimposing the corresponding sections from two models, and measuring the differences in area, changes in swelling can be estimated.

CALIBRATION OF THE APPARATUS

A. Radial distortion

Distortion and geometrical quality of the pictures obtained with this equipment were determined by the grid method (Hallert 1964, p. 360) for the greatest enlargement used. The results are presented in Figs. 4 and 5. In summary it may be said that the distortion was not significant in comparison with its standard error. The accuracy of the image co-ordinates may be described by means of the standard error of unit weight according to the method of least squares, based on the calibration of the six outer orientation elements as unknown parameters. The root-mean-square value of the standard error of unit weight was found to be 11 μm. Therefore it is possible to use this equipment to study, with high quality, the geometric characteristics of objects with areas of the order of a few square centimetres and height differences of up to one centimetre.

B. Fiducial marks

In the evaluation outlined, the inner orientation elements representing the data of the central perspective were assumed to be known. These may be determined by a special calibration procedure. Since the cameras used had originally no fiducial marks, four marks were added in such a way that each

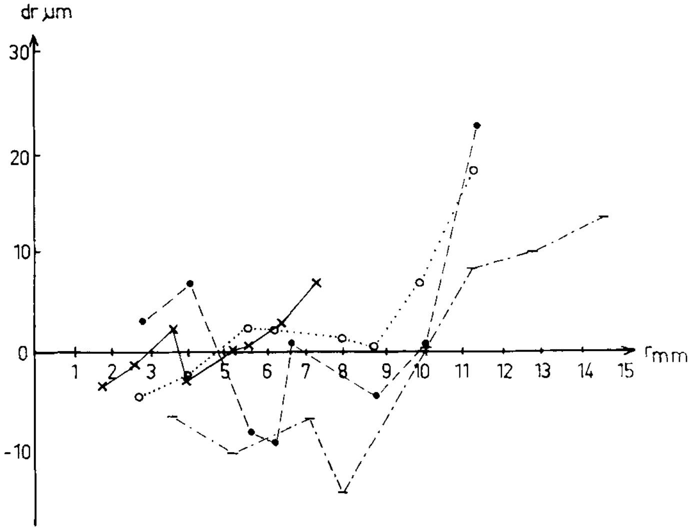

Fig. 4. Radial distortion (dr) in μm as a function of the radius (r) in mm. Determination according to the grid method. Every spot in the diagram corresponds to an adjustment by least squares of observations in points having the same radius.

×, ● left camera

—, o right camera

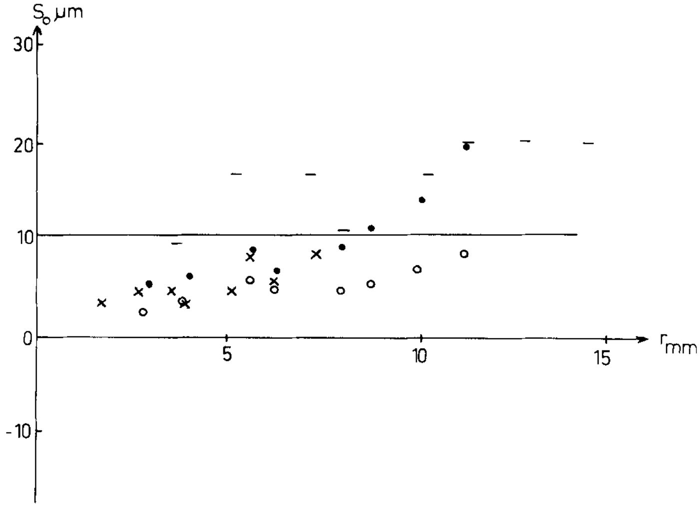

Fig. 5. Standard error of unit weight (S_0) in μm as a function of the radius (r) in mm obtained in the adjustments to determine the radial distortion (fig. 4). The root mean square of all $S_0 = 11.4\,\mu$m

×, ● left camera

—, o right camera

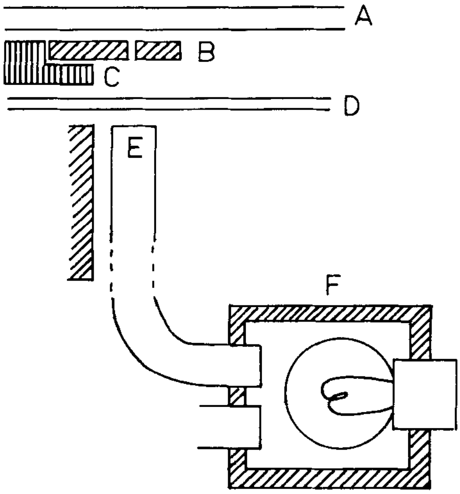

Fig. 6. Illumination of fiducial marks

A = Film
B = Metal plate
C = Hole in metal plate
D = Shutter
E = Light conductor
F = Lamp housing

corner of the picture had one illuminated mark. Each fiducial mark consisted of an illuminated hole in a metal plate which lay against the film (Fig. 6). Illumination was effected through four light conductors communicating with a light source, the intensity of which could be varied to compensate for varying times of exposure.

C. Photogrammetric calibration

If photogrammetric methods are to be used, the camera equipment must be calibrated. This implies determination of the inner and outer orientation elements of the system, and for this purpose a special test object was (Fig. 7)

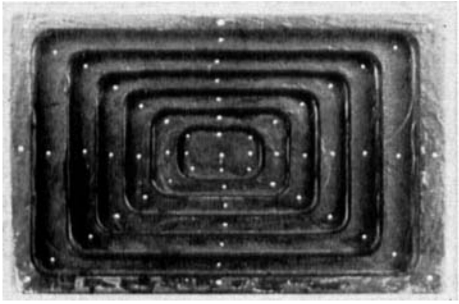

Fig. 7. Test object

developed. Outer orientation indicates the position of the camera in relation to the photographed object. Inner orientation means the position of the picture in relation to the inner projection centre. Stereophotography yields two pictures with different outer orientations, and possibly different inner orientations as well. By relative orientation is meant the outer orientation of one picture with respect to the other.

The method of calibration used was based on the relations between the co-ordinates of certain points on the photographed object and the co-ordinates of the corresponding image points. These relations are based on certain assumptions such as the existence of an outer and an inner projection centre. Thus each ray from the object is considered to be a straight line to the outer projection centre and to continue, parallel to itself, from the inner projection centre to the photographic emulsion, which is assumed to lie in a plane containing the fiducial marks which define the image co-ordinate system (x', y') and include the position of the inner projection centre. The orthogonal projection of the inner projection centre on the image plane is termed the principal point and denoted by the co-ordinates (x'_o, y'_o) in the image co-ordinate system. The perpendicular distance from the inner projection centre to the image plane is termed the image constant, or the camera constant when it is a question of a camera. The camera constant is denoted by c. Object points are denoted by co-ordinates X, Y, Z. The outer projection centre has co-ordinates X_o, Y_o, Z_o in the object co-ordinate system. The rotations of the image co-ordinate system in relation to the object co-ordinate system are denoted by ω (primary), φ (secondary), $\varkappa$ (tertiary). The relations may be written as follows:

$$x = x_o + c\frac{(X)}{(Z)} \qquad\qquad y = y_o + c\frac{(Y)}{(Z)}$$

where

$$\begin{aligned}(X) &= (X-X_o)\cos\varphi \cos\varkappa + (Y-Y_o)(\sin\omega \sin\varphi \cos\varkappa + \cos\omega \sin\varkappa) - \\ &\quad - (Z-Z_o)(\sin\omega \sin\varkappa - \cos\omega \sin\varphi \cos\varkappa)\\ (Y) &= -(X-X_o)\cos\varphi \sin\varkappa + (Y-Y_o)(\cos\omega \cos\varkappa - \sin\omega \sin\varphi \sin\varkappa) - \\ &\quad -(Z-Z_o)(\cos\omega \sin\varphi \sin\varkappa + \sin\omega \cos\varkappa)\\ (Z) &= -(X-X_o)\sin\varphi + (Y-Y_o)\sin\omega \cos\varphi + (Z-Z_o)\cos\omega \cos\varphi\end{aligned}$$

By means of a Taylor series the above formulas may be linearized using approximate values for ω, φ, $\varkappa$, X_o, Y_o, Z_o, x'_o, y'_o, and c.

By photographing points, the positions of which are known, and then measuring the corresponding image co-ordinates, a series of equations can be formed, the solution of which will give the unknown elements of orientation. In the present study there were nine unknown elements, and therefore a minimum of nine equations, i.e. at least five points were required. These five points must be situated in such a way that, if three of them lie in the same plane, the other two must lie outside that plane such that the outer projection centre does not lie on the line produced by any two of the points. As a rule there are a large number of points on the test object. If all points are located, thus yielding more than the required nine equations, a redundant system is obtained which may be solved by the method of least squares.

D. Test object

A special test object was constructed for the photogrammetric calibration of the Zeiss stereomicroscope. This consisted of a rectangular parallelepiped, in one side of which a step-shaped depression was formed (Fig. 7). The test object, made of brass, was oxidized. Points on this test object were created by drilling holes 0.3 mm in diameter through the oxidation. Eight such points were placed on each of the seven steps, which together with a central point on the lowest plane gives 57 points in all.

The points were located in a three-dimensional rectangular co-ordinate system. The measurements were made by a method described by *Torlegård* (1966, 1967).

The co-ordinates of points on the test object were determined twice, using different orientations. The two determinations were then transformed to a common co-ordinate system, using an orthogonal transformation. It should be noted that in one determination three readings were made on each point to increase the precision, while in the other only one reading was made. All points were used for the determination of the parameters of the transformation. The standard error of unit weight in this transformation was 0.0084 mm.

E. Inner orientation

The test object described above was photographed by using the Zeiss stereomicroscope. The following negative scales were used 1:0.45, 1:0.7, 1:1.1, 1:1.8, and 1:2.75.

Five stereoscopic test pictures were measured in a stereocomparator, which in this case was used as a mono comparator. The instrument used was the Wild StK 824. A description of this instrument may be found in modern textbooks of photogrammetry, for example *Hallert* (1964). This instrument is of very high quality, as demonstrated by *Hallert* (1962, 1964, 1968).

All calculations were carried out on an electronic computer. For a more detailed description of the calculation procedure reference may be made to *Torlegård* (1967).

After transformation of the comparator co-ordinates in an arbitrary image co-ordinate system, the following final fiducial mark co-oordinates are obtained which then define the image co-ordinate system.

Table I

Fiducial mark co-ordinates. Average values and standard deviation. Unit values are given in mm.

	Point	Camera 1 (4 pictures)		Camera 2 (5 pictures)	
		x	y	x	y
	1	17.0426	10.1917	17.0470	9.8314
Average value	2	−16.7785	10.1593	−16.9421	9.8314
	3	−16.0165	−9.3237	−17.1901	−9.9989
	4	16.7527	−10.0273	17.0851	−9.7697
Standard deviation of average value	1	.0014	.0011	.0013	.0012
	2	.0009	.0014	.0009	.0017
	3	.0008	.0008	.0006	.0019
	4	.0010	.0008	.0012	.0013

Because of a gross identification error of one of the fiducial marks only four pictures from camera 1 were included. The root mean square of the standard deviation of the average value was 0.0012 mm. For any particular co-ordinate the standard deviation was 0.0026 mm. If future use is made of these fiducial mark co-ordinates and it is assumed that the picture is subjected to a uniform scale change as the only deformation, it may be expected that the transformation of the fiducial marks will have a standard error of unit weight of

$$s_o = \sqrt{0.0012^2 + 0.0026^2} = 0.0029 \text{ mm.}$$

For the ten test pictures the following values were found for the inner orientation (Table II).

The standard errors in Table II calculated as the product of the standard error of unit weight and the square root of the weight coefficients, depend on the geometrical conditions which existed at the instant of exposure. In this case they were unusually large because of the very small aperture angle

Table II

Inner orientation. Unit values are given in mm.

Negative Scale	Point	*Camera 1*		*Camera 2*	
		Value	Standard error	Value	Standard error
1:2.75	x'_0	—7.142	1.520	5.467	1.080
	y'_0	3.444	1.721	—0.806	1.292
	c	120.381	4.561	111.247	2.237
1:1.1	x'_0	—19.881	2.885	15.438	3.933
	y'_0	17.373	2.962	—6.068	4.977
	c	315.600	10.042	315.986	9.098
1:0.7	x'_0	—47.973	11.503	—23.900	15.600
	y'_0	37.317	10.650	—45.486	16.784
	c	636.525	111.122	775.385	85.240
1:0.45	x'_0	—30.958	19.819	147.171	149.580
	y'_0	—36.279	21.307	26.444	31.114
	c	605.472	135.937	1495.489	1341.288
1:1.8	x'_0	—35.700	43.986	134.890	61.149
	y'_0	90.789	52.140	—65.882	32.778
	c	747.159	251.259	723.694	291.202

of the lens and the very short focusing distance. In other words, the determination of the inner orientation was weak because of the unfavourable geometrical conditions. This was also clearly shown in the matrix of the weight coefficients of the parameters determined in the adjustment. For camera 1, and a negative scale of 1:2.75, the weight coefficient matrix was as shown in Table III.

The inner orientation elements were closely correlated with certain outer orientation elements: x'_0 with X_0, y'_0 with Y_0 and c with Z_0. This means that a change in the inner orientation is to a certain degree compensated by a change in the outer orientation.

The bundles of rays for the two pictures of a stereopair were essentially the same. Because of this it was desirable that the two pictures should have the same camera constant, while the principal points must be different. From Table II it may be seen that the camera constants for the two pictures often differed and that the principal points were often situated outside the picture. For the photogrammetric measurements for which this apparatus will be used, it would be more simple, from a practical point of view, to have the same inner orientation for both pictures. Small differences would cause no serious difficulties, but the magnitude of the differences for the cameras

Table III

Weight coefficient matrix

	ω	φ	$\varkappa$	X_0	Y_0	Z_0	x'_0	y'_0	c
ω	1.0000	0.0388	0.5660	—0.0366	—0.3131	—0.0424	—0.0452	0.9962	0.0312
φ	0.0388	1.0000	—0.1486	0.0652	—0.0077	—0.1193	—0.9774	0.0381	0.0856
$\varkappa$	0.5660	—0.1486	1.0000	—0.0264	—0.3309	0.0056	0.1429	0.5516	—0.0063
X_0	—0.0366	0.0652	—0.0264	1.0000	—0.4915	0.9069	0.1471	—0.0830	—0.9110
Y_0	—0.3131	—0.0077	—0.3309	—0.4915	1.0000	—0.5297	—0.0971	—0.2293	0.5350
Z_0	—0.0424	—0.1193	0.0056	0.9069	—0.5297	1.0000	0.3107	—0.0924	—0.9993
x'_0	—0.0452	—0.9774	0.1429	0.1471	—0.0971	0.3107	1.0000	—0.0543	—0.2782
y'_0	0.9962	0.0381	0.5516	—0.0830	—0.2293	—0.0924	—0.0543	1.0000	0.0815
c	0.0312	0.0856	—0.0063	—0.9110	0.5350	—0.9993	—0.2782	0.0815	1.0000

used here introduces certain complications. The question then arises as to whether one may allow the inner orientations of two pictures to be fixed in such a way that they have the same camera constant for the same enlargement and that the principal point always coincides with the origin of the image co-ordinate system. If the above conditions are assumed in calculating the elements of outer orientation, a certain part of the approximation is compensated for in the values calculated, while the remainder causes an increase in the standard error of unit weight in the adjustment.

If the increase is great, the procedure should not be used. The standard errors of unit weight both with and without the approximation are shown in Table IV. The following equation relating quadratic forms may be written:

$$Ql = Qy + Qi + Qr$$

where

Ql is the quadratic form of the discrepancies before adjustment
Qy » » » » » » » of outer orientation
Qi » » » » » » » of inner orientation
Qr » » » » » » residual errors in the adjustment of outer and inner orientation together.

The standard error of unit weight in the first adjustment was $S_{01} = \sqrt{\frac{Qr}{n-9}}$ and in the second $S_{02} = \sqrt{\frac{Qi + Qr}{n-6}}$. Thus Qi may be found if S_{01} and S_{02} are known. The ratio was formed:

$$F = \frac{Qi}{3} \frac{n-9}{Qr}$$

If this value is not greater than the 5 % value for the F-distribution with 3 degrees of freedom in the numerator and n—9 degrees of freedom in the denominator, the assumed approximations may be considered acceptable. The F-values are shown in Table IV. In one case the F-value lies between 5 % and 1 %. The conclusion is that the assumed approximations were acceptable.

Table IV

Increase of s_0 when inner orientation parameters were not determined

Camera	Negative scale	Determined s_0	(df)	Not determined s_0	(df)	F	test
	1:2.75	.0095	(57)	.0108	(60)	0.260	not signif.
	1:1.1	.0183	(55)	.0228	(58)	3.41	almost signif.
Left	1:0.7	.0206	(29)	.0244	(32)	0.230	not signif.
	1:0.45	.0391	(17)	.0421	(20)	1.460	not signif.
	1:1.8	.0462	(19)	.0425	(12)	0.675	not signif.
	1:2.75	.0082	(67)	.0092	(70)	2.72	not signif.
	1:1.1	.0183	(57)	.0208	(60)	2.61	not signif.
Right	1:0.7	.0213	(35)	.0209	(38)	0.880	not signif.
	1:0.45	.0234	(15)	.0288	(18)	2.97	not signif.
	1:1.8	.0335	(9)	.0523	(12)	2.69	not signif.

For this approximation it was assumed that the principal point coincides with the origin of the image co-ordinate system, and the camera constants were the values shown in Table V.

Table V

Camera constants

Negative scale	1:2.75	1:1.1	1:0.7	1:0.45	1:1.8
c mm	116	316	700	1000	735

F. Outer orientation

Using the same computer program (*Torlegård*, 1967), the values of outer orientation were calculated. By subtracting the values of outer orientation of the second picture from those of the first, the relative orientation values were determined. These are shown in Tables VI and VII. The distance between the projection centres is called the stereobase. For scale 1:1.1 the base is 29.536 mm.

Table VI

Values of outer and relative orientation. Unit values are given in mm and μrad

Outer orientation		First picture	Second picture	Relative value	*Camera constant*
		11	21	11—21	
	ω	8981	15729	—6748	116
	φ	—4925	—112155	107230	
1:2.75	$\varkappa$	53486	29	53457	
	X_0	28.754	.634	28.120	
	Y_0	14.642	14.618	.024	
	Z_0	—278.095	—275.819	—2.276	
		12	22		
	ω	13298	14672	—1374	316
	φ	—3983	—107226	103243	
1:1.1	$\varkappa$	50674	2654	48020	
	X_0	29.404	—.100	29.504	
	Y_0	18.606	18.885	—0.179	
	Z_0	—289.895	—288.521	—1.374	
		13	23		
	ω	13479	14364	—885	
	φ	3965	—103523	107488	700
1:0.7	$\varkappa$	20986	740	20246	
	X_0	29.705	—10.064	39.769	
	Y_0	16.951	17.331	—.380	
	Z_0	—419.245	—395.858	—23.387	
		14	24		
	ω	29930	24834	—2904	1000
	φ	165	—104380	104215	
1:0.45	$\varkappa$	60078	11097	48981	
	X_0	31.275	—5.987	37.262	
	Y_0	12.627	11.832	.795	
	Z_0	—357.379	—356.637	.742	
		15	25		
	ω	16457	16688	—231	735
	φ	—12812	—121418	108606	
1:1.8	$\varkappa$	55190	6273	48917	
	X_0	27.027	—1375.560	28.403	
	Y_0	12.461	12.657	—.196	
	Z_0	—264.792	—263.547	—1.245	

Table VII

Standard error in outer and relative orientation. Unit values are given in mm and µrad.

Negative scale	1:2.75	1:1.1	1:0.7	1:0.45	1:1.8
ω_1	1190	1065	2658	5460	4223
φ_1	1198	1087	2676	5449	4160
$\varkappa_1$	547	468	688	1255	1630
X_0	.329	.311	1.118	1.945	1.097
Y_0	.326	.305	1.111	1.948	1.114
Z_0	.148	.133	.290	.447	.436
ω_2	741	830	1291	4398	5187
φ_2	728	833	1299	4443	5094
$\varkappa_2$	399	425	454	829	1979
X_0	.199	.238	.514	1.591	1.347
Y_0	.202	.237	.511	1.575	1.372
Z_0	.106	.121	.180	.297	.531
$\delta\omega$	1400	1348	2956	7005	6695
$\delta\varphi$	1400	1368	2978	7035	6580
$\delta\varkappa$	678	633	824	1504	2565
δX_0	.384	.392	1.229	2.515	1.738
δY_0	.384	.386	1.223	2.505	1.737
δZ_0	.182	.180	.342	.537	.687

ANALYSIS AND DISCUSSION OF THE ACCURACY OF THE METHOD

By *precision* is meant the closeness together of measurements. Precision is ordinarily expressed as standard deviation of one measurement or of the average of several repetitions. By *accuracy* is meant the closeness of measurements to the truth, to rigid conditions or defined standards. Accuracy is expressed as root mean square error (discrepancy), standard error of unit weight or standard error. See also *Hallert* (1968).

This experimental work was based on three stereomodels of a patient. The three picture pairs were taken immediately after each other, the image scale being 1:1.1. No changes in swelling took place, thus this study represents an empirical determination of the combined effects of the photogrammetric errors in the procedure. According to Table V the camera constant c = 316 mm when the scale of the negative is 1:1.1. Since the aim was to obtain a graphical plot in the stereoautograph Wild A7, the pictures had first to be reduced so that the picture constant was small enough to allow the A7 to be used for the construction of a model. Diapositives were made to a scale of approximately 2:3, giving a picture constant equal to approximately 210 mm. The reduction was determined by measuring the distances between the fiducial marks on the diapositives. These were compared with the values given in Table I, from which the picture constant was determined. With the inner orientation established in the A7, the outer orientation was restored using the values found in Table VI. The stereomodel was reconstructed in a scale of 1:1. No disturbing vertical parallaxes were found in the models, indicating that both the calibration and the construction of the model were excellent. For the graphical representation an 8:1 enlargement was used between the drawing table and the stereomodel.

The graphical plotting was done in the following way. Contours of the teeth, cavities and fillings were drawn in an x—y plane approximately perpendicular to the optical axis of the camera (Fig. 8). Five distinct reference points were chosen and marked on the teeth. Their heights (h) above the x—y plane were measured for use as reference values when absolute orien-

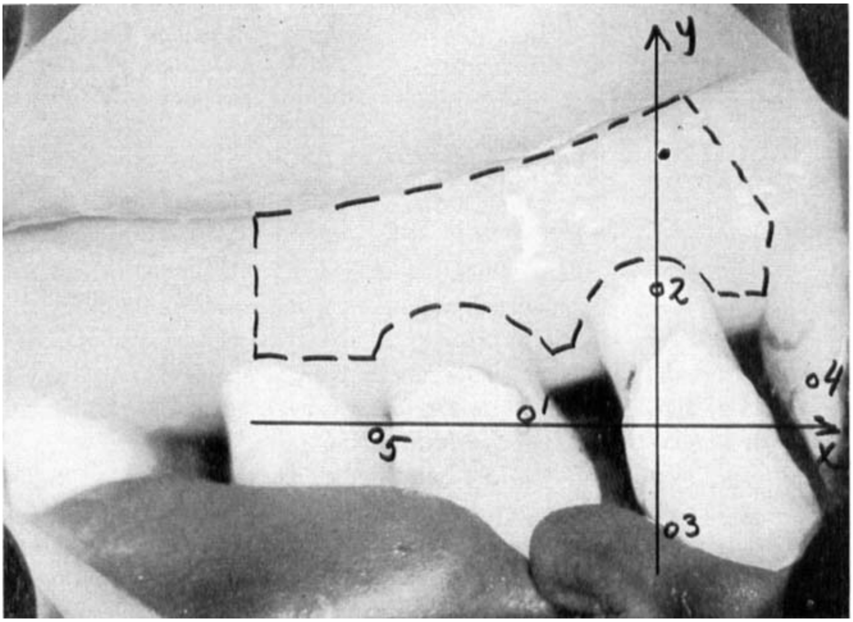

Fig. 8. Teeth and marginal periodontium 1+ . . . 4+. The registration has been made within plotted area. The x—y plane is perpendicular to direction of photography. 1, 2, 3, 4, and 5 are reference points on teeth.

tation between subsequent models and the system of the first model is desired. For differences in values of y, equal to one millimetre, x—h sections were plotted in order to show the geometry in the direction of the optical axis of the camera (Figs. 9 and 10). After the fitting of the stereomodels to the reference points, the procedure was repeated for the two remaining models.

The area of the gingiva used for comparison is shown in Figure 8. The comparison between the two graphical representations was done in two ways. First h-values were derived by using a grid with lines spaced at 1 mm. Thus at each point we have three h-values, h_{i_1}, h_{i_2}, h_{i_3}. The average of these

$$h_i = \frac{1}{3}(h_{i_1} + h_{i_2} + h_{i_3})$$

and the variance

$$s_i^2 = \frac{1}{2}\sum_j (h_{ij} - h_i)^2$$

A standard deviation in height is then calculated for the entire area

$$s = \sqrt{\frac{1}{n} \cdot \sum s_i^2}$$

where n is the number of points.

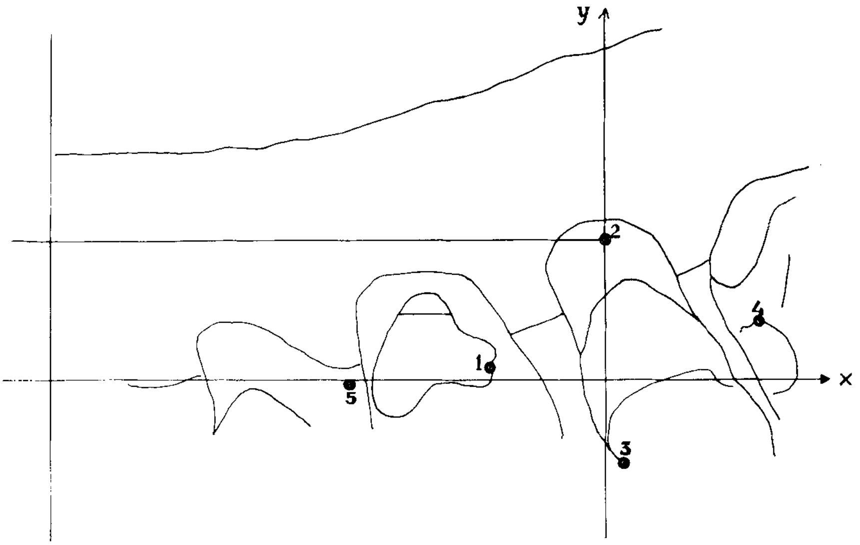

Fig. 9. Graphical representation from fig. 8. The points 1, 2, 3, 4, and 5 are reference points on teeth.

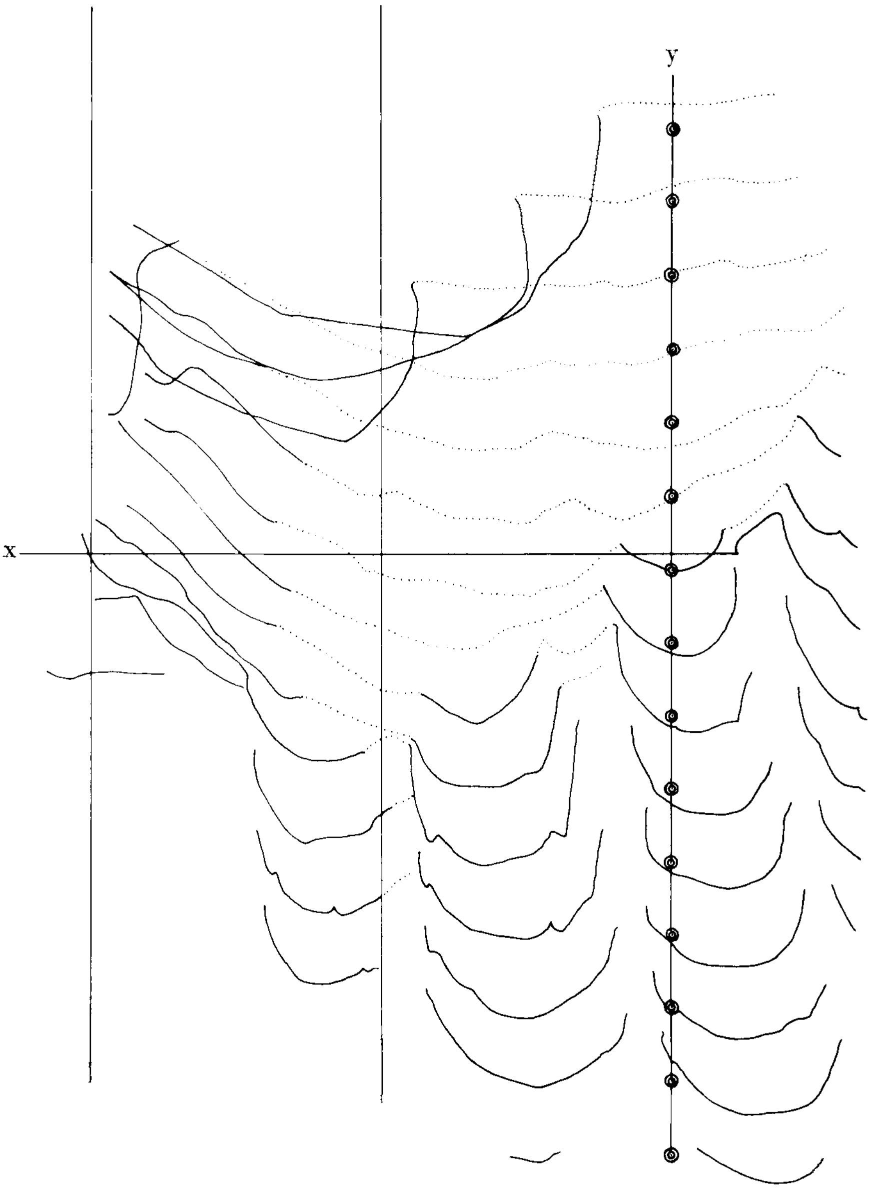

Fig. 10. Graphical recording of gingival level. The equidistance between y-values is 1 mm.

This was found to be 0.293 mm, with $2 \times 90 = 180$ degrees of freedom.

Secondly, the three drawings of the sections were superimposed, using the reference points. Between the sections of the first and second model there is a long narrow area which represents the difference between the two. For each section this area was measured with a polar planimeter. The area was considered positive if the first curve was outside the second, otherwise negative. Since the distance between the sections was 1 millimetre, the change in volume was simply the sum of the differences in area of all the sections. The difference between the sections of the first and third models was calculated in the same manner. The volumes, thus obtained, were 15.25 mm^3 and 35.70 mm^3. These values apply to an area of 92 mm^2 and are differences from an unknown volume. Thus three volumes, V_1, V_2, V_3 ($V_1 = 0$) ,can be determined. The standard deviation for an estimate of the volume then becomes

$$s = \sqrt{\frac{1}{2}\left(V_1^2 + V_2^2 + V_3^2 - \frac{(V_1 + V_2 + V_3)^2}{3}\right)}$$

$$s = 17.85 \text{ mm}^3$$

For purposes of comparison this may be converted to a height distance by dividing it by the total area. This yields

$s = 0.194$ mm, with 2 degrees of freedom.

This value, at the 5 % level of significance, was not significantly different from the value obtained earlier from measurement of heights.

The calibration data according to Table IV (Neg. scale 1:1.1) were

$$s_0 = 20.8\,\mu\text{m and } s_0 = 22.8\,\mu\text{m}$$

the root mean square being 21.8 μm. The model co-ordinates, recorded here, had a standard error consisting of two components, partly derived directly from the image co-ordinates and partly, indirectly, a result of the standard error of the points used in the absolute orientation. A simple, but approximate rule for the direct effect of the image co-ordinates is that the x—y co-ordinates of the model have a standard error which is the standard error of unit weight multiplied by the scale factor, while the height co-ordinates have a standard error which may be derived by dividing the x—y standard error by the ratio of the stereobase to the object distance. For a scale factor of 1:1 and a ratio of 1:10 the direct effect from the image co-ordinates is

$$s_h = 0.218 \text{ mm}$$

The effect of the absolute orientation may be determined using the general law of error propagation. We may begin with the differential formula for height in absolute orientation:

$$dh = dh_0 - x d\eta + y d\xi$$

where dh : discrepancy in height
dh_0 : general translation in height
$d\eta$: rotation about the y-axis
$d\xi$: rotation about the x-axis
x, y, : the co-ordinates of points with respect to the centre of gravity of the points used for absolute orientation.

The quantities dh_0, $d\eta$ and $d\xi$ are determined from the discrepancies in the points used for absolute orientation. In this case the weight coefficient matrix for these parameters is shown in Table VIII.

Table VIII.
Weight coefficients for the points used in absolute orientation

	dh_0	$d\eta$	$d\xi$
dh_0	0.20000	0	0
$d\eta$	0	0.012116	0.003517
$d\xi$	0	0.003517	0.047393

To get the standard errors in the parameters from these values one must know the standard error of unit weight for the absolute orientation S_0H. From a purely theoretical point of view this should be directly dependent on the standard error of the image co-ordinates. Thus $S_0H = s_h = 0.218$ mm. The discrepancies, dh, which are the basis for the determination of S_0H are not the differences between the measured and given (true) values, but rather between the measured values in different models. After carrying out the absolute orientation the co-ordinates of the five points used for this orientation are as shown in Table IX.

From the variation in the values shown in Table IX the standard deviation in the points used for the absolute orientation may be calculated with nine degrees of freedom:

$$S_H = 0.017 \text{ mm}.$$

This is highly significantly different from the theoretical value of 0.218 mm, which is probably due to the standardized procedure used for the photography and the correlation between the different models thereby introduced.

Table IX

The co-ordinates of the reference points after absolute orientation. Unit values are given in mm.

Point	Model 1	Model 2	Model 3	Average Value
1	10.000	10.000	10.050	10.017
2	10.460	10.300	10.460	10.397
3	10.666	10.700	10.660	10.675
4	9.910	—	9.850	9.880
5	6.036	6.000	6.030	6.022

To the differential formula for absolute orientation we now apply the general law of propagation of the standard errors (actually, the weights) for that portion of the model for which the changes in volume are to be determined. The area is represented by 91 points, for which the weight coefficients for the effect of absolute orientation are determined. The average of these weight coefficients is

$$QHH = 1.944$$

If the area had had a more regular shape the average could have been more easily calculated by integration as follows:

$$QHH = \frac{1}{Y}\iint(0.2 + x^2\ 0.012116 + y^2\ 0.047393 - 2xy\ 0.003517)\,dxdy$$

where Y is the area measured.

The standard error in the heights after absolute orientation within the area under consideration can now be determined in three different ways from the formula

$$sH = \sqrt{Sh^2 + QHH \cdot S^2oH}$$

Alternative 1:

$$sh = 0.218\text{ mm}$$
$$so = 0.128\text{ mm}$$

Both the direct and indirect effects are determined from the standard error of unit weight:

$$sH = 0.374\text{ mm}$$

Alternative 2:

$$sh = 0.218\text{ mm}$$
$$so = 0.047\text{ mm}$$

The direct effect is obtained from the standard error of unit weight of the image co-ordinates and the indirect effect from the discrepancies in the points used for the absolute orientation:

$$s_H = 0.228 \text{ mm}$$

Alternative 3:

$$s_h = 0.047 \text{ mm}$$

$$s_o = 0.047 \text{ mm}$$

Both the direct and indirect effects are obtained from the points used for the absolute orientation:

$$s_H = 0.081 \text{ mm}$$

The value found empirically for s_H was 0.293 when determined by pointwise measurement and 0.196 when determined by planimetry. This indicates that Alternative 2 gives the best determination. It is also clear that the effect of the image co-ordinates is best determined from their standard error, while the effect of absolute orientation is best determined from the variation in the points used for the absolute orientation, which may be found from Table IX.

SUMMARY

A Zeiss stereomicroscope with low magnification was used. Stereoscopic pictures could be taken by simultaneous observation through the oculars. The cameras used were not constructed for measuring purposes, thus fiducial marks had to be added. Further, all movable parts were sealed in order to secure the inner geometry of the camera. Photogrammetric calibration was done by photographing a special test object followed by measurements and calculations of the inner and outer orientation. During the exposure the patient was fixed to the camera system by means of a biting plate.

A Wild A7 stereoautograph was used for measurement and plotting. A preliminary study using three models of the gingiva indicated that the method can be used to determine the level of the surface of the labial gingiva with a standard error of 0.2—0.3 mm. Differences in volume were readily calculated using a planimetric method in connection with graphically represented sections.

The obtained values of accuracy refer to changes of the gingival level in the direction of photography. Changes perpendicular to this direction could be estimated with a standard error that was about one tenth of the above mentioned. Since pathological changes also occur in these directions, the accuracy of the determination of such changes can be expected to be considerably higher than the accuracy of the determination of changes in the depth direction.

RÉSUMÉ

MODIFICATIONS DE VOLUME DU TISSU GINGIVAL. ETUDE STÉRÉOPHOTOGRAMMÉTRIQUE

L'appareil utilisé est un stéréomicroscope Zeiss à faible grossissement. Des photographies simultanées peuvent être prises à travers les oculaires. Les appareils photographiques utilisés ne sont à l'origine pas construits pour des fins de mensuration; c'est pourquoi ils ont été munis de marques internes en treillis. De plus, toutes les parties mobiles ont été rendues fixes pour assurer la géométrie intérieure de l'appareil photographique. Le calibrage photogrammétrique a été obtenu par la photographie d'un objet de test spécial, avec ensuite mensurations puis calcul des éléments de l'orientation intérieure et extérieure. Pendant la prise de vue, le patient est maintenu en position par rapport à l'appareil photographique au moyen d'une plaque d'occlusion. Cette pratique a facilité la mise au point et l'estimation des variations de volume.

Comme instrument de travail, nous avons utilisé un stéréoautographe Wild A7. Une série d'épreuves préliminaires de trois stéréomodèles de la gencive indique que la méthode peut déterminer le niveau de la surface du tissu gingival vestibulaire avec une erreur standard de 0,2—0,3 mm. Les différences de volumes peuvent être calculées facilement par une méthode planimétrique en utilisation des représentations graphiques des coupes.

Les degrés de précision obtenus concernent les modifications du niveau du tissu gingival dans la direction de la prise de vue. Les modifications perpendiculaires à cette direction peuvent être déterminées avec une erreur standard qui est environ le dixième de l'erreur mentionnée ci-dessus. Puisque les modifications pathologiques se font aussi dans ces autres directions, on peut s'attendre à ce que ces altérations puissent être déterminées avec une précision bien supérieure à celle avec laquelle on détermine les modifications dans le sens de la profondeur. L'appareil décrit semble ainsi convenir à l'étude des modifications de volume de la gencive dépassant 0,2 mm dans la direction de la prise de vue, et 0,02 mm dans une direction perpendiculaire à celle-ci.

ZUSAMMENFASSUNG

VOLUMVERÄNDERUNGEN DES ZAHNFLEISCHGEWEBES. EINE STEREOPHOTOGRAMMETRISCHE UNTERSUCHUNG

Es wird über die Anwendung eines Stereomikroskops in der Zahnheilkunde nach ZEISS mit schwacher Vergrösserung berichtet. Stereoskopische Aufnahmen werden bei gleichzeitiger Beobachtung durch das Okular aufgenom-

men. Da die Kammern nicht für Messungen vorgesehen waren, wurden sie vorher mit eingebauten Rahmenmarkierungen ausgerüstet. Die beweglichen Teile innerhalb der Kamera wurden plombiert, um jegliche Verschiebungen auszuschliessen und die innere Orientierung sicherzustellen. Photogrammetrische Kalibrierung geschieht durch Bildaufnahmen an einem besonderen Testobjekt mit nachfolgenden Messungen und Berechnungen von inneren und äusseren Orienterungen. Während der Aufnahme ist der Patient mittels einer speziellen Beissplatte an das Kamerasystem fixiert. Dadurch wird die Fokuseinstellung und die folgende Berechnung des Schwellungsgrades des Zahnfleiches erleichtert.

Als Auswertungsinstrument wurde der Stereoautograph WILD A7 angewandt. Eine Versuchsserie an drei Stereomodellen des Zahnfleisches hat ergeben, dass man mit dieser Technik die Niveaudifferenzen der labialen Gingivaoberfläche mit einem mittleren Fehler von 0,2—0,3 mm bestimmen kann. Volumenänderungen können leicht mit der planimetrischen Methode in den untersuchten Bezirken errechnet werden.

Es mag ausserdem darauf hingewiesen werden, dass die erhaltenen Genauigkeitswerte in der Richtung Kameraobjektiv — Objekt gelten, Änderungen senkrecht zu dieser Achse betragen nur ein Zehntel der oben angegebenen Werte. Da pathologische Veränderungen im Zahnfleisch sich nach allen Raumrichtungen ausbreiten, bedeutet es, dass der mittlere Fehler der registrierten Veränderungen unterhalb der mitgeteilten Werte liegt.

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