An investigation of gingival topography in man by means of analytical stereophotogrammetry

I. Methodological aspects

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The use of a camera for metrical purposes presupposes a knowledge of errors affecting the photograph. The aim of the present study was to test stereomodels of defined test objects with regard to inherent deformations. Three different test objects were used, a fine-lined grid and two test models resembling the teeth and the surrounding labial gingiva. The accuracy as given by the standard error of unit weight of absolute orientation was 0.018 mm (precision grid), 0.066 mm (convex surface) and 0.115—0.139 mm (jaw model). The precision under the test conditions as judged from the comparison of stereomodels from repeated exposures of the same object (jaw model) was between 0.009 and 0.032 mm. Since the stereomodel was deformed in a similar way irrespective of the test object used, it was concluded that part of the total error could be interpreted as a systematic deformation of the stereomodel. For further *in vivo* application of the method, with difficulties to identify reference points on the gingiva from repeated exposures, a simple way to estimate the accuracy of gingival measurements was derived by extrapolation from the standard error of the reference points on the teeth.

Key-words: Photography; gingiva; microscopy

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Using photogrammetry it is possible in many cases to determine the size, form and position of an object, in all three dimensions in space, from measurements made on photographs of the object. Generally at least two different projections (photographs) of the object are necessary. From each such stereopair of photographs a three dimensional model of the part of the object appearing in both photographs may be constructed. This may be done by means of stereovision in instruments of very high quality especially constructed for the purpose of making stereophotogrammetric measurements. Such measurements must be based on the geometrical centre or principal point of each photograph, which point must be defined within suitable tolerance limits (*Hallert*, 1964).

In an earlier published article (*Berghagen, Bergström & Torlegård*, 1968) a photographic method devised for the purpose of recording the appearance and form of the gingiva under *in vivo* conditions was described. The camera system was calibrated. As a part of this calibration the interior geometry and radial distortion of each of the cameras were determined. The system was found to be of such a quality that it could be used for photogrammetric measurement purposes.

Where measurements are to be made for the purpose of determining changes in the relative positions of points on an object, it is of considerable advantage to have reference points of which the relative positions do not change during the period used for the study. In photogrammetry such reference points should be welldefined object points which are easily identifiable on each of the stereopair of photographs. From the point of view of accuracy of measurement of points which undergo change, it is of advantage to have the reference points well distributed over the object. Such a requirement is, however, difficult to fulfil when measurements are to be made on gingival tissue since this tissue is not of a stable nature and does not have a surface structure with easily identifiable points from stereopair to stereopair of photographs. To minimize this difficulty, reference points may be chosen on the surface of the teeth in the immediate neighbourhood of the gingival area to be studied.

The purpose of the present study was to determine the effect of the number and distribution of the reference points on the accuracy and precision with which the positions of other object points could be determined. Of special interest was that case where the object points to be determined lay outside the area of the reference points.

MATERIAL AND METHODS

The photographic apparatus consisted of a stereomicroscope and two cameras (Fig. 1). The microscope, of two parallel optical systems, was equipped with a common auxiliary lens. Each optical system was equipped with a camera, which meant that the object was imaged simultaneously in two different perspectives. In other words, the photographs were two different projections of the object. A built-in lamp, (6 V 50 W) was used for illumination of the object during exposure. The exposure time was 1/30 sec. Kodak EHB-135 colour film, of format 24 × 36 mm, was used. This film gave good resolution for the illumination used. The object could be viewed during exposure, since a portion of the light could pass through the semitransparent mirrors to the oculars, making well focussed, sharp photographs more easily possible.

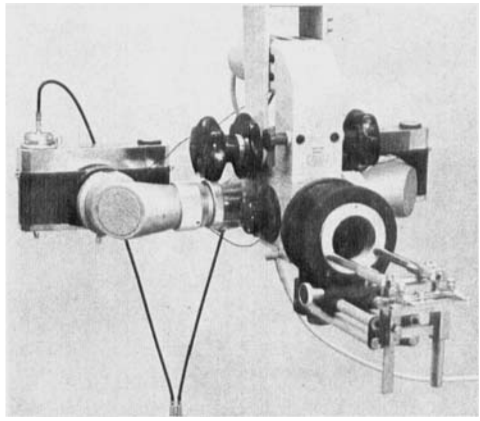

Fig. 1. Stereomicroscope with cameras, object-holder and focussing bar.

Each camera was equipped with four fiducial marks in the form of small holes, one in each corner of the film holder, to locate the principal point. During exposure each hole was illuminated, and thus imaged on the film. The distance between fiducial marks in the film plane was 33 mm in the length, or x, direction and 20 mm in the width, or y, direction. Calibration of the photogrammetric camera system has been described in an

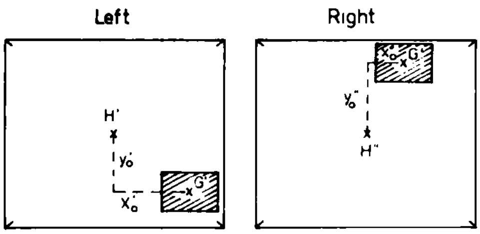

Fig. 2. Position of films and fiducial centers of photograph (G',G") in relation to principal points (H', H") in left and right cameras respectively.
$x_0' =$ 47,9 mm $x_0'' = 23.9$ mm
$y_0' = —37,3$ mm $y_0'' = 45.5$ mm

earlier published work (*Berghagen et al.*, 1968). The position of the principal point, H, with respect to the photograph center, G, as defined by the fiducial marks, is shown in Fig. 2. The camera constants, C_1 and C_2, were each equal to 770 mm

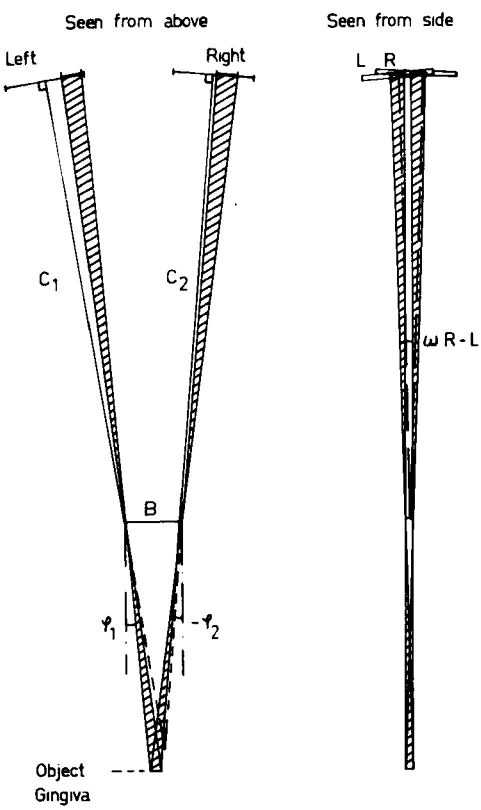

Fig. 3. Schematic drawing of the geometric relations among cameras and object. The angles of rotation were $\varphi_1 = 11.7^g$, $\varphi_2 = —6.1^g$ and $^{w}RL = 3.8^g$.

and the negative scale was 1:0.67. The photogrammetric case is shown in Fig. 3. The angles of convergence were $\varphi_1 = 11.7^g$ and $\varphi_2 = —6.1^g$. During photography the object was fixed to a holder in a stable and well focussed position. The position of the holder relative to the microscope could be adjusted in all three directions. Focussing or adjustment in the depth direction was done with the aid of an adjustable bar which is shown in Fig. 1.

Test objects

1) *Grid.* A precision grid, referred to as R 102, and with line spacing of 0.631 mm in the x-direction and 0.609 mm in the y-direction was used to test for lack of flatness. The co-ordinates of the points of intersection on the grid were known to 0.002 mm. In the calculations the given co-ordinates of the points of intersection were considered free from error. A total of 156 intersection points were measured in the stereomodel. Five well distributed intersection points were used to define the reference plane.

2) *Test models.* Two different test models were used in those studies conducted to determine how well the stereomodel reproduced the convex surface of the mucous membrane of the gingiva.

Test model I was made of acrylic and constituted a convex surface resembling a portion of the labial gingiva. This surface covered an area of 6.340 × 6.763 mm in the xy-plane. The maximum difference in depth was 0.501 mm. Thirteen measuring points were embedded in the acrylic surface. The measuring points were made of steel wire with a mean cross section area of 0.119 mm (S.D. 0.003), and contrasted well with the surrounding material.

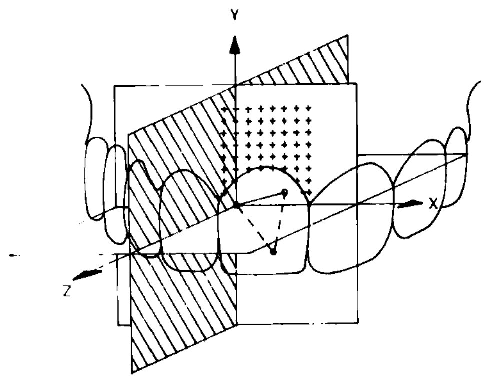

Fig. 4. The three-dimensional axis-system defined on test model II.

Test model II was a plaster cast of an upper jaw and represented teeth and gingiva in normal size and shape (Fig. 4). This analysis extended over an area corresponding to the medial incisors and the gingiva situated apically thereto. The gingival surface was represented by a thin layer of wax. Measurement points of good contrast, 0.352 mm in diameter

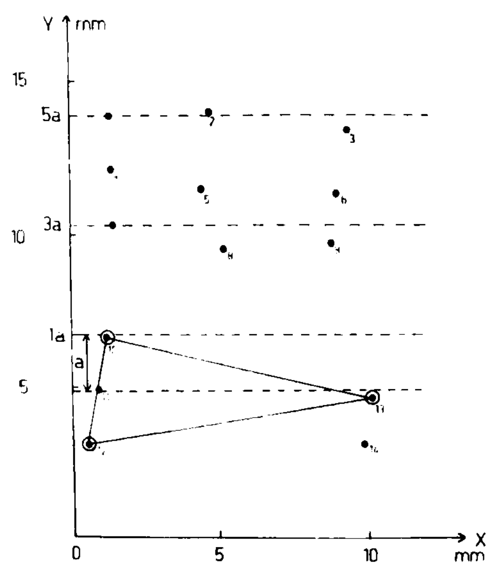

Fig. 5. The distribution in xy-plane of gingival points 1—9 and dental points 10—14 on test model II. The points 10, 12 and 13 were used as reference points in the case where absolute orientation was performed using three dental points.

(S.D. 0.035), well distributed over the area to be studied, were embedded in this layer. Nine such points, »gingival points», and five easily identifiable points on the surfaces of the two incisors, »dental points», were used (Fig. 5). The size of the surface area was 10.040 × 13.521 mm in the xy-plane. The maximum difference in depth (z) was 2.898 mm. Ten exposures of this model were made. Between each two exposures the model was removed from its fixed position in the bite plate, the bite plate was loosened from its holder and the whole was reset.

Determination of the co-ordinates of the reference points in the test models

A Wild stereoautograph A7 equipped with a stereomicroscope (Nife Micrall) was used to determine the relative positions of the measuring points in the test models. Details of the method have been described by *Torlegård* (1966, 1967). The accuracy attained was 0.007 mm in the xy-plane and 0.013 mm in the depth, z, direction. Each point was measured three times and the x-, y-, and z-co-ordinates were read and recorded for each measurement. The average values were regarded as error-free and accepted then as the given or known values of the point-co-ordinates in all calculations. The precision of a single measurement, based on the three repeated settings and expressed as standard deviation was $S.D._x$ = 0.003 mm, $S.D._y$ = 0.004 mm, and $S.D._z$ = 0.009 mm. These values are of the same order of magnitude as those reported by *Torlegård* (1966).

Resolution

Resolution, from negatives, was done in a Wild StK 824 stereocomparator, a first order instrument of exceptionally high quality (standard error of unit weight,

$s_0 = 0.9$ μm; *Hallert*, 1964). In this instrument a subjective optical model is obtained from the simultaneous viewing of the stereopair of photographs. A built-in floating mark allows the operator to simultaneously and stereoscopically measure point co-ordinates on both of the photographs. The space co-ordinates of the object points measured were then calculated.

The stereocomparator makes use of orthogonal co-ordinate systems. The read-out comprises co-ordinates in the left photograph (x′, y′) and the differences between the right and left photograph co-ordinates, i e the horizontal parallax P_x, and the vertical parallax P_y. The measurements were made using 11 x magnification. The read-out is automatic and was recorded on punched cards. All calculations were performed in an IBM 360/75 computer.

Photogrammetric measurements of stereopairs

As stated in the introduction, from each stereopair of photographs a three dimensional model of the part of the object appearing in both photographs may be constructed. To understand how this is possible let us first consider the photography case. Here our »model» is the object itself. Considering a single »point» on the object, and using ideal central projection, this point is imaged on each of the photographs by straight lines, or rays, emanating from the point and passing through the respective projection centres of the cameras, which may be arbitrarily positioned relative to each other for the photography. The distance between the projection centres, is the base B and the direction of each of the cameras, defined by the line from the principal point to the projection centre, may be described by the angles of rotation about each of the three space directions x, y, z. These angles may be termed ω_1, φ_1, $\varkappa_1$ for the left camera and ω_2, φ_2, $\varkappa_2$ for the right camera (Fig. 6).

If we now consider the reverse of the photography case, that is the construction

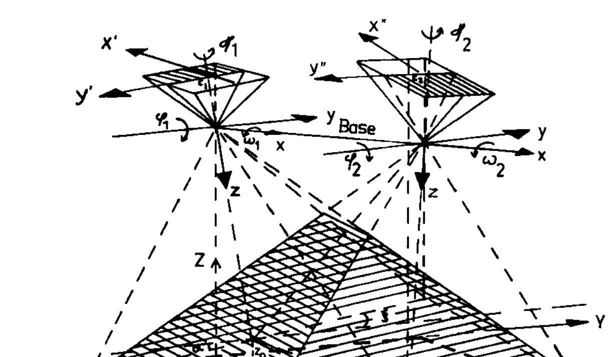

Fig. 6. Drawing showing concepts of relative and absolute orientation.

of a model of the object from the two photographs, we must place the photographs in the same position relative to each other as they had during photography so that the two rays which emanated from the object »point» will intersect in space and create the corresponding model point. In other words the direction angles or angles of rotation must be found.

Knowing the angles of rotation at the time of exposure a scaled model may be constructed the scale being linearly proportional to the assumed base. Also the space co-ordinates x, y, z of any point on the model may be found since these are simple functions of the base, the direction angles, and the image co-ordinates of the point on the two photographs x′, y′ on the left photograph and x″, y″ on the right photograph:

$$x = B \cdot f_1 (\varphi_1\ \varkappa_1\ \omega_2\ \varphi_2\ \varkappa_2\ x'\ y'\ x''\ y'')$$
$$y = B \cdot f_2 (\varphi_1\ \varkappa_1\ \omega_2\ \varphi_2\ \varkappa_2\ x'\ y'\ x''\ y'')$$
$$z = B \cdot f_3 (\varphi_1\ \varkappa_1\ \omega_2\ \varphi_2\ \varkappa_2\ x'\ y'\ x''\ y'')$$

Note that ω_1 does not appear in the above relations. This is because we can always use that value of ω_2 which corresponds to $\omega_1 = 0$.

Concepts of relative and absolute orientation

The relative orientation is the procedure by which the five direction angles (φ_1, $\varkappa_1$, ω_2, φ_2, $\varkappa_2$) are found by forcing five corresponding pairs of rays to intersect. When the relative orientation is performed with more than the minimum number (5) of corresponding pairs of rays, a perfect fit will not result due to small errors in measurement and identification and to the fact that our mathematical formulae are not a perfect representation of reality. This misfit caused by such redundancy is interpreted as a measure of the accuracy of the system and expressed as a standard error of unit weight for relative orientation

$$s_o = \sqrt{\frac{\sum v^2}{n-q}}$$

where v is the residual, or lack of equality, in each orientation observation equation
n is the number of equations, or number of corresponding pairs of rays
q is the number of unknowns. Here q = 5

If the base is known, the calculated space co-ordinates x, y, z of all points will produce a model of the object of the same size as the object. If the base is not known it may be found by using any assumed base and comparing some calculated distance d on the model

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

with the corresponding known distance on the object. Hereby the model scale is determined.

The so constructed stereomodel is a scaled replica of the object, as nearly as the accuracy of the measurement will allow, and in general lies in a system of space co-ordinates non coincident with that of the object space co-ordinate system. In order to facilitate comparisons of successive stereomodels taken over a period of time it is desirable to transform one system to the other. The transformation in space through which the axes of the two co-ordinate systems are brought into coincidence is executed using three translations x_o, y_o, z_o and three rotations ξ, η, α about the space axes X, Y, Z respectively. Absolute orientation consists of the above space transformation plus the determination of the model scale and therefore requires the seven parameters

x_o, y_o, z_o, ξ, η, α and the scale factor. To determine these seven parameters a minimum of seven object co-ordinates must be known. (Not all co-ordinate combinations may be used.) If more information is available, the redundancy will give rise to a standard error of unit weight for the absolute orientation

$$s_o = \sqrt{\frac{\sum v^2}{n-q}}$$

where v is the residual, or lack of equality, in each absolute orientation observation equation
n is the number of equations, or number of known object point co-ordinates
q is the number of unknowns. Here q = 7.

The mathematical relations used for relative orientation, and absolute orientation, as well as the expressions for the calculations of space co-ordinates, may be found in any standard textbook of photogrammetry, for example *Hallert* (1964).

Definition of precision and accuracy

According to photogrammetric error theory (*Hallert*, 1964) the concept of precision denotes the closeness together of measurements and is expressed by the standard deviation of a single measurement or of the mean. By accuracy is meant the closeness of measurements to defined standards or given (true) values. It may be expressed by the root mean square (r m s) of discrepancies between direct measurements and given quantities. For indirect measurements of unknown quantities the standard error of unit weight (s_o) after the adjustment according to the method of least squares is an expression of the basic accuracy. After the least squares adjustment there will in general, always remain residual errors or residuals.

RESULTS

Relative orientation

It was evident both visually, through viewing of the stereopairs, and numerically, from the magnitude of residual parallaxes, that the quality of the relative orientation was good. The values of the standard error of unit weight (s_o) based on fifteen points, or fifteen pairs of corresponding rays, lay between 0.008 mm and 0.020 mm in the scale of the photograph for the various models.

Absolute orientation

Grid

It was possible to perform the absolute orientation, based on five points with each of x, y, z co-ordinates known for each point, with such accuracy that the standard error of unit weight s_o was 0.018 mm. The differences in height from the known values of the grid plane are shown in Fig. 7. It is seen that the stereomodel of the grid plane is deformed compared to

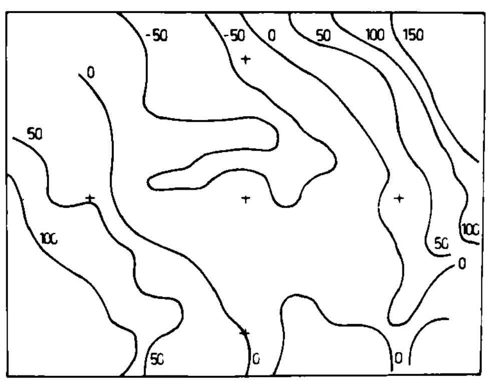

Fig. 7. Topography of the deformed stereomodel showing discrepancies from the grid plane (μm).

reality, the central part being the bottom of a concavity. The root mean square values of the components of the discrepancies were

$$\mathrm{rms}\, s_x = 0.025 \text{ mm}$$
$$\mathrm{rms}\, s_y = 0.016 \text{ mm}$$
$$\mathrm{rms}\, s_z = 0.060 \text{ mm}$$

Test Model I

The standard error of unit weight of the absolute orientation, based on thirteen points, was

$$s_o = 0.066 \text{ mm}$$

In this case the calculated height co-ordinates were of the same quality as the plane co-ordinates.

Test Model II

The standard error of unit weight of the absolute orientation, based on all fourteen points, for the ten repetitions was such that

$$0.115 \text{ mm} < s_o < 0.139 \text{ mm}$$

It was considered to be of value to study the magnitude and distribution of the residuals in the gingival points in the case where the absolute orientation was based on only three dental points compared with the case where all five dental points plus nine gingival points were used (Fig. 5).

When the absolute orientation was done with only three points the root mean square values of the discrepancies for the gingival points were

$$\mathrm{rms}\, s_{x2} = 0.125 \text{ mm}$$
$$\mathrm{rms}\, s_{y2} = 0.171 \text{ mm}$$
$$\mathrm{rms}\, s_{z2} = 0.298 \text{ mm}$$

In the case where they were included in the orientation process the root mean square values of the residuals for the nine gingival points were

$$\mathrm{rms}\, s_{x1} = 0.093 \text{ mm}$$
$$\mathrm{rms}\, s_{y1} = 0.120 \text{ mm}$$
$$\mathrm{rms}\, s_{z1} = 0.200 \text{ mm}$$

Statistically, however, using the F-test for comparison there was no significant difference between the two sets of figures; $F_x = 1.81$, $F_y = 2.03$, $F_z = 2.22$ ($P > 0.05$, df $= 8$).

Another reason for this study was to obtain information on how faithfully stereopairs of photographs could be repeated. By using clinical-like conditions it was hoped that the study would give some idea as to be reproducibility of the method. The agreement between the relative positions of the various points in the ten different models became a measure of precision in the measurement and the registration procedure. Expressed as the standard deviation for a single measurement and treating x-, y- and z-co-ordinates separately according to

$$\mathrm{S.D.} = \sqrt{\frac{\Sigma v_i^2}{n_1(n_2-1)}},$$

where $v_i = x_i - \bar{x}$, $y_i - \bar{y}$ and $z_i - \bar{z}$, respectively,
$n_1 = 14$ (number of points),
$n_2 = 10$ (number of repetitions),

the precision of the calculated co-ordinates when the absolute orientation was performed with fourteen points in each of the ten models was

$$\mathrm{S.D.}_x = 0.009 \text{ mm}$$
$$\mathrm{S.D.}_y = 0.010 \text{ mm}$$
$$\mathrm{S.D.}_z = 0.020 \text{ mm}$$

The corresponding values of the precision when the absolute orientation was performed with only three points was

$$\mathrm{S.D.}_x = 0.011 \text{ mm}$$
$$\mathrm{S.D.}_y = 0.021 \text{ mm}$$
$$\mathrm{S.D.}_z = 0.032 \text{ mm}$$

DISCUSSION

The standard error of unit weight of absolute orientation indicates the accuracy with which the stereomodel, determined from photograph co-ordinates, can be fitted to the three-dimensional object which it represents, based on corresponding points in both systems. For that case where the points used for the absolute orientation (reference points) lie in one portion of the model while the measurement points of interest lie in another portion of the model the basic accuracy in this latter portion may not be known with the same accuracy.

Theoretically, the magnitude of the error in the portion lying outside the portion containing the reference points may be estimated by linear extrapolation from the standard error of unit weight of the absolute orientation. Of especial interest is the propagation of error in the z, or depth, direction. Assume that the distance between two points, x_1 and x_2 with known accuracy, is 2a and that the distances of the points from a given plane are h_1 and h_2 respectively (Fig. 8).

The distance h_p from the given plane, of a point x_p, distant x from the midpoint of x_1 and x_2 may be expressed as follows:

$$h_p = h_2 \left(\frac{x + a}{2a}\right) + h_1 \left(-\frac{x - a}{2a}\right)$$

If h_1 and h_2 can be measured independently and have a standard error $m_1 = m_2 = m$, the standard error of h_p

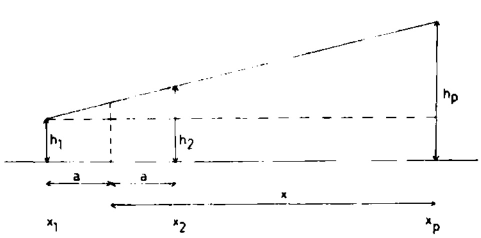

Fig. 8. Derivation of extrapolation factor.

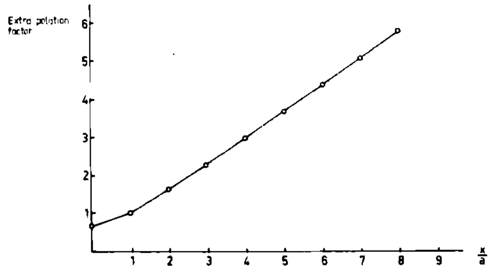

Fig. 9. Extrapolation factor function.

which may be found form the special error propagation law is

$$m_p^2 = m^2 \left(\frac{x + a}{2a}\right)^2 + m^2 \left(\frac{x - a}{2a}\right)^2$$

$$\text{and } m_p = m \sqrt{\frac{\left(\frac{x}{a}\right)^2 + 1}{2}}$$

The factor $\frac{m_p}{m}$ indicates how the basic accuracy in the position of a measured point is affected by its distance from the center of the reference points. In Fig. 9 is shown how the factor increases by this distance. It should be noted that the standard error of the extrapolated point x_p is dependent also on the distance, *a* in the direction of extrapolation, between the reference points.

This mathematical model has been tested in test model II, for the case where the absolute orientation was based on only three dental points. In this case the root-mean-square value of the discrepancies of the z-co-ordinates for the nine gingival points was $rms_z = 0.298$ mm.

Using the reference system shown in Fig. 5 an extrapolation factor for each of the nine gingival points with regard to its y-distance from the centre of the reference system was calculated. The average of the nine calculated values was 3.02, and hence as an average $m_p = m \cdot 3.02$ mm. For $m_p = rms_z = 0.298$ a standard error in depth (m) for the reference points of 0.099 will be found.

This value may be compared with the empirically obtained value of rms_z = 0.071 mm of the residuals in depth for the dental points in that case or with the values of rms_z = 0.060 mm and 0.066 mm found from the tests with the grid plane and test model I respectively.

These investigations show, then, that in the case of *in vivo* application, where the reference system consists of dental points, the determination of gingival points lying outside the reference system may have somewhat less accuracy than indicated by the standard error of unit weight of absolute orientation.

The investigations also give some indication that the stereomodel is deformed (test for lack of planarity with grid) and that this deformation is in part systematic (repeated exposures giving similar deformation, test model II). It is probable that the greatest portion of the model deformation derives from lack of planarity of the film and film shrinkage. The lens system may also be considered as a source of error, although no significant radial distortion was evident from the calibration (*Berghagen et al.*, 1968). Model deformation resulting from imperfect relative orientation (*Hallert*, 1956) is judged to have less effect since the model, as a result of a moderate lens angle, covers a very limited field (Fig. 2). The effect of the deformation of the model on the accuracy of the measured points depends in part on the reproducibility of the outer orientation and in part on the nature of the deformation.

For further use of the method in vivo it may be reasonably assumed that errors caused by this deformation in part remain the same from stereomodel to stereomodel and that their effect therefore in part will disappear when comparisons by differences are made.

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