

Stress distribution in diametral compression tests

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In an attempt to find an explanation for an experimentally found strong connection between the extension of the loaded area and the load required for fracture, an analysis of the diametral compression test was carried out by means of the finite element method. The distribution of the stresses in a quadrant of a circular specimen section was studied for varying contact surface but at a given load distribution within this surface. Below a limit value for the width of the contact surface tensile stresses appeared peripherically outside the contact area, which had a magnitude exceeding those stresses which are centrally located and should normally initiate fracture. For that reason, peripheral fracture initiation must be regarded as a possibility. Only marginal importance could be attached to changes in the extension and magnitude of the central tensile stresses, as compared with the potential impact of the peripheral tensile stresses.

Key-words: Finite element analysis; tensile test; dental materials

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The diametral compression test is a method used for measuring the tensile strength of brittle materials. In odontology, this method is widely used when testing materials such as amalgam, cements, ceramics and plaster products.

In testing according to the diametral compression method the specimens are shaped as linear, circular cylinders which are loaded diametrically between two plane-parallel platens. This generates maximum tensile stresses in and transversely to the loaded diametral plane. These stresses are proportional to the applied load and should, under adequate experimental conditions, initiate the fracture. If the specimen's

areas, which are loaded by the platens, are too small the fracture may, in accordance with previous assumptions, be initiated instead by the large compressive and shear stresses that occur under the loaded area (4). The pattern of cracks which then appears should, however, differ from that which appears in the case of valid tensile fractures (4).

In areal contacts the maximum tensile stresses are equal within a central section of the diametral plane. The size of this section, and thus the amount of material which is exposed to the maximum tensile stresses, is negatively correlated to the loaded area (3). However,

according to statistical theories (5) there is also a connection (a negative correlation) between the loaded amount of material and the apparent strength, a phenomenon which has served as the explanation of an experimentally found positive correlation between loaded area and strength (1).

From previous experimental work (2) it was concluded that the apparent strength was conditioned by the size of the loaded area to such an extent that the previous statistical explanation was felt to be questionable. Therefore a closer study of the correlation between loaded area and stress distribution in diametral compression tests seemed justified. The purpose of the present work is, consequently, to analyze by means of the finite element method the distribution of the stresses as a function of the size of the load bearing area.

MATERIALS AND METHODS

The stress distribution in diametral compression tests was studied by means of the finite element method (FEM). This is a well-known method of calculating stresses in loaded structures (6). In the FEM the structure in question is divided into numerous small elements. These are assumed to be interconnected at a discrete number of nodal points on their boundaries. This provides a connection between the deformations and the stresses in adjacent elements. The division into elements results in a great number of mutually similar correlations, which makes computerization possible.

For symmetrical reasons it was possible to limit the FEM-analysis to a quadrant of the circular specimen cross section (Fig. 1). It was assumed that the specimen was loaded via relatively soft strip-shaped paddings of varying

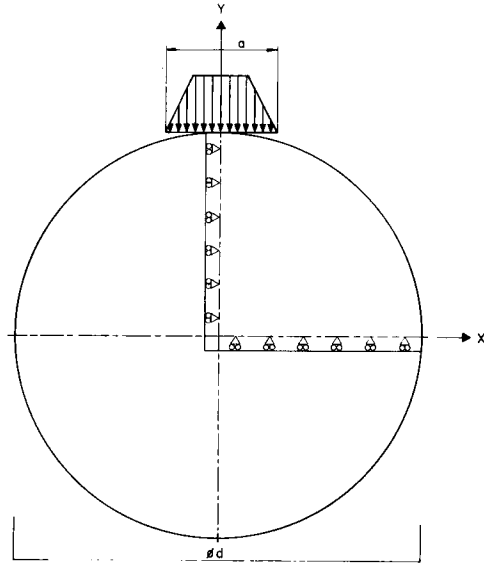


Fig. 1. The distribution of the load in the loaded area (a) and an analyzed quadrant.

widths in accordance with a method previously used (2). For the calculations the following assumptions were made: specimen length 1, radius 100, elasticity module 1 and Poisson's number 0.25. The load 100 was distributed along the loaded area as shown by Fig. 1. The normal stresses were calculated for the nodes, which are marked by the transpositions of the lines in the quadrant shown by Fig 2a. Five different load cases were analyzed. In the various load cases (I-V) the loaded area was limited by the nodes indicated in Fig. 2 b. Thereby the width of loaded area (a) to diameter (d) ratio was varied from a/d 0.025 to 0.15 (Table 1).

In order to establish the correlation between the maximum value of the central tensile stresses and load bearing area the arithmetical mean was calculated for the nodal points within the central fifth of the loaded diameter. The extension of the maximum tensile stresses along the loaded diameter was calculated as the part of the diameter within which the tensile stresses deviate

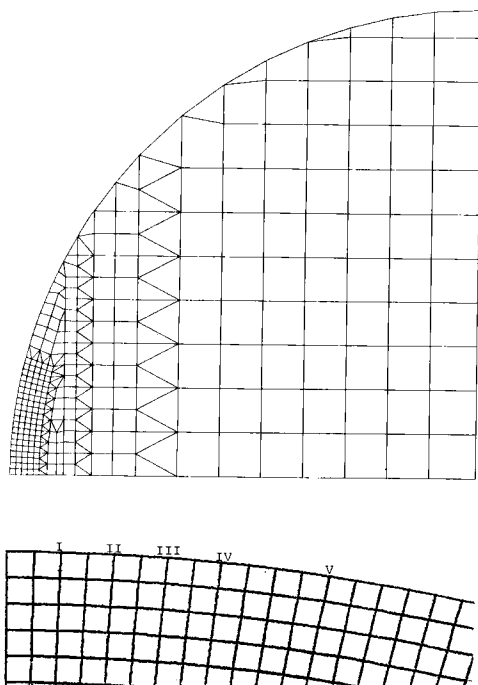


Fig. 2. Quadrant divided as to elements (Fig. a) and the extension of the loaded area for each specific load case. (Fig. b).

from the maximum value by less than 10 %.

RESULTS

The greatest central tensile stresses appeared for the lowest a/d (width of loaded area to diameter ratio). Fig. 3 shows that these tensile stresses were reduced by 0.9 % at an increase of a/d from 0.025 to 0.1. At a/d 0.15 the reduction was 2.3 %.

Fig. 4 shows the distribution of the tensile stresses along the loaded diameter. Table 2 shows the extension of those stresses which differ by less than 10 % from the maximum value for each specific load case. For the load case I, II, III and IV the extension amounts to 65–85 % and for load case V to 45 %, of the loaded diameter. The extension of

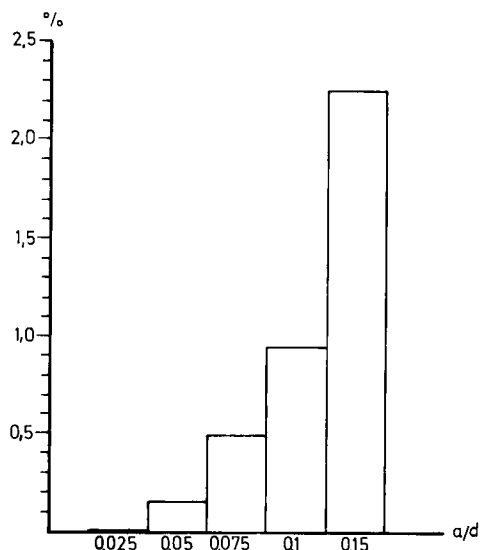


Fig. 3. Reduction of the central tensile stresses (as a percentage), when a/d increases from 0.025.

Table 1. Width of loaded area to diameter ratio in the different load cases

Load case	a/d
I	0.025
II	0.05
III	0.075
IV	0.1
V	0.15

Table 2. Extension of maximum tensile stresses along the loaded diameter

Load case	Extension of stresses (percent of diameter)
I	85
II	75
III	65
IV	65
V	45

Table 3. Maximum compressive stresses in the load bearing area

Load case	Compressive stress
I	31.97
II	14.48
III	9.296
IV	6.825
V	4.467

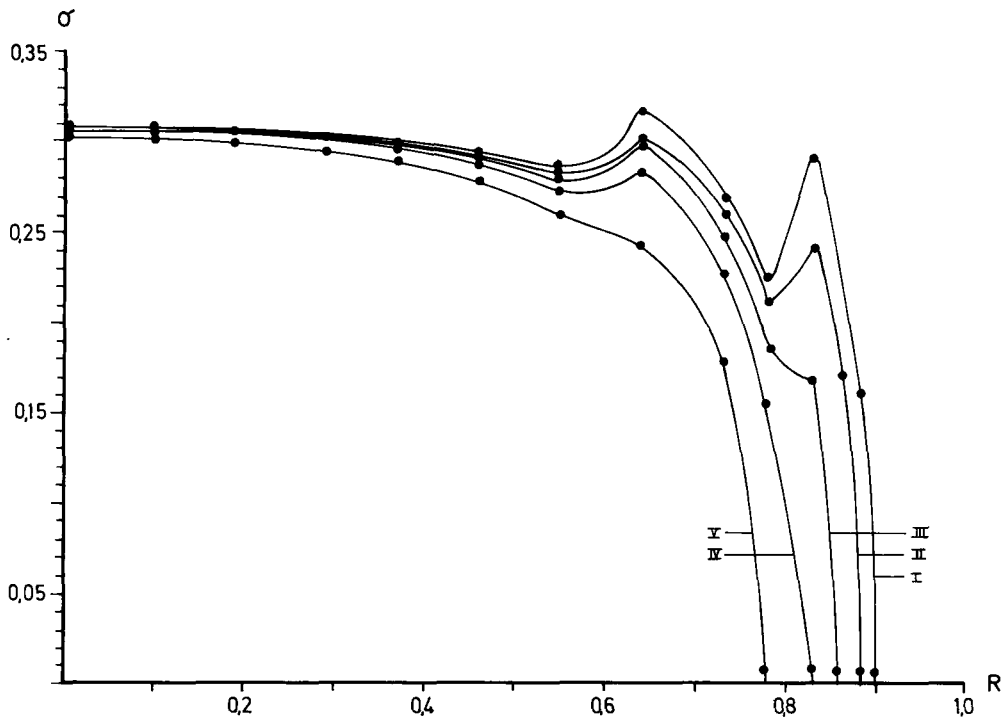


Fig. 4. Tensile stresses along the loaded radius (R) for the various load cases (I - V). $R = 0$ corresponds to the centre of the loaded area.

stresses which differ by less than 5% from the maximum value was estimated, for load case V at 37% of the loaded diameter.

Great tensile stresses (exceeding 0.30) appeared, except along the loaded diameter, also along the periphery (load case I, II). The greatest peripheral tensile stresses appeared throughout for the node immediately outside the contact area.

Fig. 5 shows the correlation between a/d and the maximum peripheral and central tensile stresses. The peripheral stresses here appear to exceed the central ones beneath a limit value for a/d which is roughly equivalent to 0.075. Fig. 6 shows, for the different load cases, the stresses for three peripheral nodes ($l = 1, 2, 3$) immediately outside the borderline of the contact area ($l = 0$). It is here obvious that the stress distinctively increased in the vicinity of

the contact area for the load cases I, II and III.

The highest compressive stresses appeared in all the load cases along the periphery in the centre of the contact area (Table 3).

DISCUSSION

The purpose was to study the test when the specimen was loaded via relatively soft strip-shaped paddings of varying widths. By the use of such paddings neither deformation nor load is guided.

In the calculations principal stresses were used as for brittle materials. Fracture will be initiated by the highest principal stresses and it will propagate at right angles. The load distribution in the contact area had to be assumed. It was, however, thought that the prob-

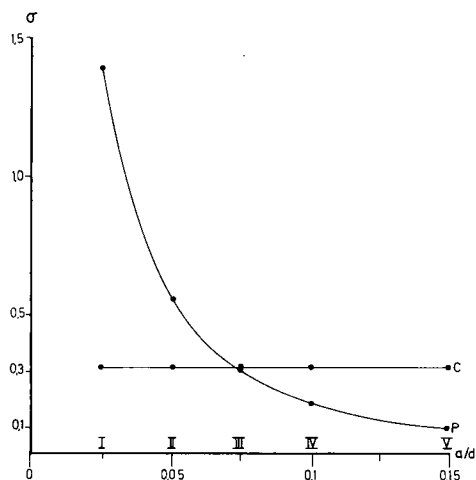


Fig. 5. Correlation between a/d and the maximum tensile stresses along the loaded diameter (C) and along the periphery (P).

able deviations from this approximation in an actual load case would only to a minor degree affect the result.

It is evident from the results that the magnitude of the maximum tensile stresses centrally are negatively correlated to a/d . As a consequence thereof the «apparent strength» increases by 0.9 %, corresponding to an increase of a/d from 0.025 to 0.1 and thereafter by 1.3 % on a further increase up to a/d 0.15.

The distribution of the load has a substantial impact on the extension of the maximum tensile stresses in the diametral plane. Between the extreme cases I and V the difference as regards extension is approximately 40 %. In the load case V the maximum tensile stresses operate along less than half the diameter. Consequently, a standardized experimental procedure as regards the extension of the contact surface and the load distribution within this surface is a prerequisite to make these tensile stresses operate along a well-defined area of the diametral plane.

In the load cases I and II tensile stresses of a magnitude definitely ex-

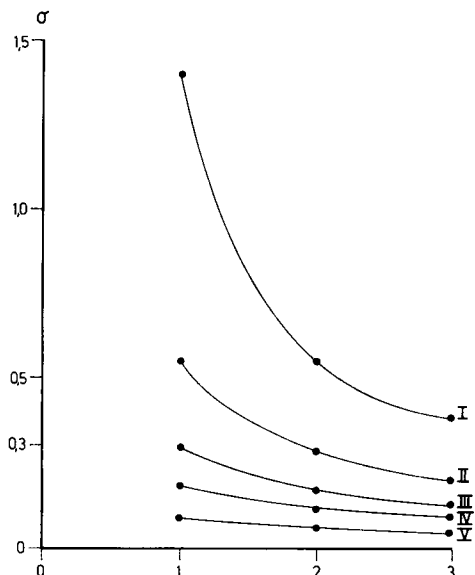


Fig. 6. Tensile stress in the periphery outside the borderline of the loaded area ($l = 0$) for load cases I-V.

ceeding the stresses in the central sections appeared for nodal points on the circle periphery close to the contact area. In the load case I they are somewhat more than 4 times greater peripherically than centrally. In the load cases III, IV and V the stresses are lower peripherically than centrally. However, in load case III that difference is very small (4 %). On account of the sparsity of the nodal points the greatest stresses registered peripherically are likely to fall slightly below the actual ones. So, the central maximum value should probably be surpassed also in load case III. From this it could be concluded that a/d under given circumstances may reach values for which tensile stresses appear, adjacent to and near the loaded area, when stresses are great enough to locate the initiation of the fracture peripherically. Moreover, small changes of a/d will then have a heavy impact on the fracture load. The corresponding impact exercised by the previously discussed changes in the

extension and magnitude of the central tensile, is probably of marginal importance.

In experiments with specimens made of dental stone it has been shown (2) that the fracture load can be strongly influenced by changes in a/d . When e.g. a/d was increased from 0,05 to 0,10 – by the use of soft, strip-shaped padings – this was reflected by an 80 % increase of the fracture load. Such a correlation between a/d and fracture load could probably not be explained as an effect of changes in the magnitude or extension of the central tensile stresses but rather as a consequence of peripheral initiation of fracture.

In order to obtain as representative a test result as possible for each specific specimen size, it is necessary to have the largest possible extension of the maximum tensile stresses, hence a low a/d . Conditioned by the given prerequisites a lower limit value for a/d appears around 0.10 via the peripheral stresses, if the approximation is carried out in view of the previously discussed circumstances in load case III.

If a minimum stress extension of 37 % (for tensions deviating by 5 % from the maximum value) along the diametrical plane is accepted, the top limit value for a/d would amount to 0.15.

Within the acceptable interval for a/d (0.10 – 0.15) the maximum tensile stresses will then differ by 1.3 % at the most. The difference between the largest and the smallest extension of the equivalent tensile stresses in the diametrical plane will then amount to 20 %, within the same interval. Meeting requests for such a narrow variation range for a/d would in most cases probably require special technical arrangements in practical tests. Moreover, it is necessary to emphasize that during such practical tests it could not be taken for granted that loading con-

ditions prevail which are identical with those assumed. So other a/d values might turn out to be more adequate. The previously quoted experiments (2) actually confirmed that higher a/d values were necessary in order to avoid a strong connection between a/d and failure load.

Summing up it could be concluded that, for previously accepted a/d values, peripheral tensile stresses may arise which are many times greater than the centrally located maximum tensile stresses, and that the extension of the latter stresses can fall well below 50 % of the loaded diameter. The load transfer techniques and their precision should consequently, when carrying out practical tests, be the object of special methodological studies where the possibility of peripheral fracture initiation should be particularly considered.

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