Supplementary material for Ringbæk TP, et al. Monte Carlo simulations of new 2D ripple filters for particle therapy facilities. Acta Oncol 2013;53:40–49.

Supplementary material

A series of superimposed Gaussian curves yield either a flat plateau or, when the distance λ of the Gaussians is not fine enough, a plateau with a ripple. It can be observed that this ripple (if the distance is not too large) is well approximated by a harmonic function (compare to Figure 1b). The understanding of this behavior is, for example, useful for the analysis of the inhomogeneities induced by a ripple filter. In relation with the raster scan technique, this oscillation was described in [9] by Weber who gives below a practical and accurate approximation for the amplitude.

The superposition of an infinite series of Gaussian functions shifted by a distance λ is given by

$$F(x,\lambda) = \sum_{i=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(x-i\lambda)^2}{2}\right) \cong 1 + A(\lambda)\cos\left(\frac{2\pi x}{\lambda}\right). \quad (13)$$

The amplitude $A(\lambda)$ can be expressed by analytical calculation of the infinite sum at $x = 0$:

$$A(\lambda) \cong F(0,\lambda) - 1 = \frac{\lambda}{\sqrt{2\pi}} \vartheta_{00}(0, e^{-\lambda^2/2}) - 1, \quad (14)$$

where ϑ_{00} is one of the Jacobi theta functions. Since the theta functions are numerically not available on most platforms, it is here developed in good approximation:

$$\vartheta_{00}(0,x) \cong 1 + 2x + 2x^4 + O_{\text{rest}}(x^{n \geq 9}), \quad (15)$$

which leads to:

$$A(\lambda) \cong \begin{cases} \frac{\lambda}{\sqrt{2\pi}}(1 + 2G(\lambda) + 2G^4(\lambda)) - 1, & \text{if } \lambda > 1.42 \\ 0, & \text{if } 0 < \lambda \leq 1.42 \end{cases} \quad (16)$$

where

$$G(\lambda) = \exp(-\lambda^2/2)$$

The absolute accuracy of Equation 13 in combination with Equation 16 and the strength of the oscillation amplitude is given by Table I. Table I shows that a superposition of Gaussians having a shift (λ) smaller than 1.6σ yields a practically flat dose. When the shift becomes larger than 2σ, the ripple amplitude A(λ) strongly increases. In a range of $\lambda < 2.7$, where the oscillation is smaller than 13.3%, the given approximation (Equations 13–16) nearly perfectly fits the exact superposition ($\Delta < 0.01\%$).

Remark: For the practical usage the formulas above can be easily scaled for Gaussian functions with $\sigma_{Gauss} \neq 1$ ($\exp(-x^2/2\sigma^2)$): $\lambda/\sigma \mapsto \lambda$.

Table I. Amplitudes of the oscillation and accuracy of the approximation.

Accuracy of Equation 13		Oscillation strength A (λ)	
		λ = 1.6	0.09%
		λ = 1.8	0.45%
		λ = 2.0	1.44%
		λ = 2.2	3.39%
		λ = 2.4	6.50%
		λ = 2.6	10.78%
λ< 2.7	Δ< 0.01%	λ = 2.7	13.30%
λ< 3.0	Δ< 0.10%	λ = 3.0	22.3%
λ< 3.6	Δ< 1.00%	λ = 3.6	44%
λ< 4.0	Δ< 3.00%	λ = 4.0	59%