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SIMPLE EXPERIMENTALLY DERIVED ALGORITHM FOR COMPUTER CALCULATED DOSE RATES ASSOCIATED WITH ^{137}Cs GYNECOLOGIC INSERTIONS

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POWERS et coll. compared the MIR (Mallinckrodt Institute of Radiology) program with three well-known programs in 1966. The three programs were by SHALEK & STOVALL (1962, 1968), ADAMS et coll. (1964) and ADAMS & MEURK (1964). Radium was used as the nuclide. The second and third programs are for intracavitary treatment and the first program for interstitial treatment only.

Their programs were prompted by the fact that a difference in dose of the order of 20 per cent at a particular point occurred. An interesting point was illustrated, i.e., the distance to be moved for normalization of the dose for the several programs is only a few millimeters. Thus, because of the steep gradients that occur in intracavitary (and even more in interstitial) implants, large per cent differences may occur for either a calculated or measured dose rate at a specified point. This led to the conclusion that accuracy is best expressed in terms of distances to be moved in order to obtain agreement between different programs rather than trying to compare measured or calculated dose rates.

Another interesting observation they made was that the maxim 'at distances from a source equal to or greater than two times the length of the source, the dose variation is nearly approximated by an inverse square function' is nearly true and can be used as a rapid approximation of dose without the necessity of performing explicit calculations in the estimation of doses from intracavitary applicators.

Finally, they suggested that an ideal program would have a simple, rapid, and accurate input; a concise and accurate computation method using reasonable factors and logic and a clearly meaningful output. This latter statement summarizes the basis for this report.

Conventional computational methods

One of the well-known methods for intracavitary dose calculations is the SHALEK & STOVALL (1968) interval method for determining dose rate to a point P from a linear ^{137}Cs source and is based upon dividing the active length into N point sources and summing the contributions from each. The basic formula for one source is:

$$(\dot{D}_p)_j = \frac{\Gamma cs A f}{N} \sum_{i=1}^{N} \left[\frac{e^{-\mu t_i}}{d_i^2} \right] \times T(d_i) \qquad (1)$$

where

$(\dot{D}_p)_j$ = the dose rate at the point p from the jth source
Γcs = the exposure rate constant for ^{137}Cs
f = rad/roentgen conversion factor for muscle tissue
A = the activity of the source
μ = the linear attenuation coefficient of the wall materials for ^{137}Cs photons
d_i = the distance from the point P to the center of the ith point source of a source
t_i = the path length within the filter for a ray from the ith point source

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$T(d_i)$ = a tissue correction factor, compensating for absorption and scatter

N = number of point sources assumed per actual active length of a single source

This formula would be applied to all sources and the results all summed for point P.

BREITMAN (1974) goes into some detail expanding upon the definition and empirical fitting of the terms mentioned and derives by computer calculation a table of dose rates for a variety of ^{137}Cs tube geometries. Active lengths are for 1.35 cm, 1.5 cm, 3.0 cm and 4.5 cm, and for a variety of filtrations and active diameters as well.

Using tables published by SHALEK & STOVALL (1968) as a basis for comparison between radium sources and cesium sources having the same construction and equivalent radium content, the dose distributions were found to be similar with the exception of the heavily filtered ends where 9 per cent difference occurred due in part to the complex spectrum of radium.

When BREITMAN's result is compared with KRISHNASWAMY's theoretical data (1972) for stainless steel encapsulation, agreement is found only on the transverse axis. KRISHNASWAMY's tables were derived from MIRD (Medical Internal Radiation Dose Committee of the Society of Nuclear Medicine) data according to BERGER (1968). The two methods agreed to within one per cent at most points when the computer method was used to generate a table for a steel-filtered cesium needle, using a linear energy absorption coefficient of 0.221 cm^{-1} based on HUBBELL's data (1960). This procedure confirmed (1) the equivalence of the two approaches to calculating dose rates and the conclusion that the dose distribution of platinum filtered ^{137}Cs sources is more closely related to that of platinum filtered ^{226}Ra sources of the same size than to stainless steel filtered ^{137}Cs sources. Actually, the first part of this conclusion was determined much earlier by HORWITZ et coll. (1964), and others, including the first author. An explicit simple power expression for dose rate was derived from a ^{137}Cs point source in an infinite unit density medium based upon the Monte Carlo derived data (BERGER) after observing that a plot of log of specific absorbed fraction versus the log of distance from the point source formed a straight line.

A least squares fit gave the expression

$$\dot{D} = \frac{k_1}{d^{k2}} \times A \qquad (2)$$

where $\dot{D}$=dose rate (Gy/h) in an infinite unit density medium at a point d cm from the point source of ^{137}Cs of A activity (in Bq) and k_1 and k_2 are constants obtained from a least squares fit.

Computer algorithm

Using the theoretically derived expression for dose rate from a point source, a simple program can be written in which a simple do loop is utilized to break the source up into any number of pieces such that each piece, i, of linear source, j, can be treated as a point source. The total dose rate to a given point of interest, p, is thus given by a simple numerical integration, i.e.,

$$\dot{D}_p = \sum_{j=1}^{M} \sum_{i=1}^{N} \frac{k_{1\,A_{ij}}}{d_{ij}^{k_2}} \qquad (3)$$

where

$$d_{ij} = [(X_{ij}-X_p)^2+(Y_{ij}-Y_p)^2+(Z_{ij}-X_p)^2]^{\frac{1}{2}} \qquad (4)$$

X_{ij}, Y_{ij}, Z_{ij} = the coordinates of the ith point source of the jth source

X_p, Y_p, Z_p = the coordinates of the point of interest

A_{ij} = the activity of the ith point source of the jth source

M = the number of linear sources

N = the number of point sources specified per each real source active length

$\dot{D}_p$ = the total dose rate at point p from all sources

Program features such as the provision to go back and easily correct coordinates that were typed incorrectly into the terminal, the ability to modify source strengths and thus optimize point A–rectal (or bladder) dose rate differentials, use of the conversational mode, the ability to print out dose rates in specified planes of interest (one centimeter actual spacings), etc., provide a useful, versatile program. Basically, the only input data required are: (1) The x, y, z coordinates at the end of each active length, (2) the activity in mCi of each source, (3) the lateral and a.p. radiographic magnification factor, (4) the source decay factor, and (5) spatial coordinates of points of interest. Data can be extracted from a set of orthogonal films, inputted into a conventional teletype terminal and outputted all in less than half an hour. The same program can of course also be implemented on a small dedicated type computer and the coordinates inputted using a digitizer.

Comparison between theoretical dose rates

A comparison of theoretical dose rates for a wide selection of clinical points of interest between the present data and those of KRISHNASWAMY agreed within one per cent on the perpendicular bisector for a single 2 cm source of active length 1.2 cm. This is of no great surprise since both were derived from the same basic MIRD data according to BERGER. Slight differences of the order of 3 per cent occurred at points not lying on the transverse axis due to the fact that the present model did not include the effects of oblique filtration although it did inherently contain the effect of 0.5 mm of platinum on the transverse axis. In an attempt to retain the same simple analytical algorithm, experimental data were obtained from a Rando-Alderson phantom and the constants k_1 and k_2 were adjusted in order to force agreement between the computer printout and the experimental data.

General experimental procedure

A ^{137}Cs source (OL=2.0 cm, AL=1.2 cm, filtration=0.5 mm platinum) was placed at approximately the external os position in the Rando-Alderson phantom. LiF (TLD 700 powder) detectors were placed in the same place of the phantom, i.e., on the perpendicular bisector of the source and in places above and below the source. Each slab was 2.5 cm thick, but powder only occupied the central 1.2 cm portion of each slab corresponding to the active length of the source. Irradiations were carried out over both a 17-hour and an 18.5-hour time interval and averages of readings were recorded. The powder was read out on a Harshaw TLD Unit and readings were used to determine doses and therefore dose rates using a calibration curve based upon the known outputs of a ^{60}Co teletherapy unit. Readings were plotted against length of the powder and the peak at the center point was taken as the correct reading for determining the dose. A contrast medium (Hypaque) was next placed into the teflon capsules which had been used to hold the LiF powder and the capsules repositioned into the phantom at the same positions which the LiF capsules had occupied. A.p. and lateral films were exposed and coordinates corresponding to the ends of the active source and centers of the Hypaque filled capsules were carefully measured from the film. (Actually, all coordinates were known by geometry, but the radiographic procedure allowed for a better simulation corresponding to the routine clinical practice.) The existing computer program was used to calculate dose rates at the measured points and these dose rates were compared with the directly measured dose rates.

Comparison of theoretical and experimental dose rates

Differences varied from 4 to 10 per cent between the experimental and computer derived data. By adjusting k_1 and k_2, differences were minimized to within 2 per cent except at extreme oblique positions. The differences within a solid angle produced by $\pm 45°$ relative to the transverse axis at the center of the source were less than 2 per cent. This is the same magnitude of difference when calculating with precisely known spatial coordinates as compared with coordinates obtained from a set of orthogonal films.

After adjusting k_1 and k_2, differences of dose rate along the transverse axis for a single source varied systematically from the data of KRISHNASWAMY; the difference being 10 per cent at a distance of 7 cm. At a distance of 2 cm, the difference was 5 per cent. In all cases the experimental (or computer generated value after adjustment of k_1 and k_2) dose rates were always lower. This is intuitively expected since the theoretical model is based on an infinite medium and the experimental data are based upon a finite medium (the Rando-Alderson phantom). As one proceeds further from the source, less scatter component would be expected in the finite medium, thus reducing the dose rate from that which would occur in an infinite medium.

Conclusion

POWERS et coll. have suggested that an ideal program would have a simple, rapid and accurate input and a concise and accurate computation method using reasonable factors and logic as well as a clearly meaningful output. The authors feel as if the present algorithm provides the basis for such a program. An analytical expression in lieu of 'table lookup' allows conciseness, reduction of required computer memory and the accuracy of the computation is assured by the agreement ($\pm 2\%$) with experimentally derived data using a female Rando-Alderson phantom. The input parameters are simple and can be quickly ascertained. Much of the accuracy depends on the individual. Spatial coordinates can be obtained directly from a set of orthogonal films by hand or can be inputted directly using a transducer. The output is meaningful and useful in afterloading techniques if one allows for the printout of dose rates from individual sources as well as the summed dose rates from all sources for any point of interest. This is useful since the source strengths can easily be modified within the computer until more desirable differential dose rates are obtained. The results are also interesting in that not only is energy build up and attenuation accounted for in the unit density medium but finiteness of the medium appears to have an effect, reducing the dose rate with distance from the source as compared with theoretically expected values. In practice, this is of no great consequence though since points of clinical interest such as points A, B, C, rectum, bladder, etc. lie relatively close to the sources.

SUMMARY

A simple mathematical algorithm is derived from experimental data for dose rates from ^{137}Cs sources in a finite tissue equivalent medium corresponding to the female pelvis. An analytical expression for a point source of ^{137}Cs along with a simple numerical integration routine allows for rapid as well as accurate dose rate calculations at points of interest for gynecologic insertions. When compared with theoretical models assuming an infinite unit density medium, the measured dose rates are found to be systematically lower at distances away from a single source; 5 per cent at 2 cm and 10 per cent at 7 cm along the transverse axis. Allowance in the program for print out of dose rates from individual sources to a given point and the feature of source strength modification allows for optimization in terms of increasing the difference in dose rate between reference treatment points and sensitive structures such as the bladder, rectum and colon.

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Copies of the computer program written in Fortran IV are available from the authors upon request.

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