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CORRECTION FOR THE ANGULAR DEPENDENCE OF A DETECTOR IN ELECTRON AND PHOTON BEAMS

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Abstract

Based on experimentally obtained angular response functions a simple analytic function has been found which can be accurately fitted to such data. Using this function formulas are derived for the variation of the detector response as the angular distribution of the electrons are changing with depth in a phantom both in electron and photon beams. The formulas show that for a flat liquid ionization chamber, a response variation of 2 to 4 per cent is normally obtained in the build-up region of electron and photon beams whereas the response is practically constant beyond dose maximum.

Most radiation detectors used for measurements in electron and photon beams have a response which is dependent on the direction of the incident beam relative to the reference or symmetry axes of the detector. Such a dependence may be due to the mechanical design of the detector or to the intrinsic angular sensitivity of the detection process. This is often so when the detector material is strongly polarized or anisotropic as is the case for flat liquid ionization chambers (6, 8) and for 'solid ionization chambers' in the form of silicon diodes (10, 15).

The angular dependence of a liquid ionization chamber is shown in the Figure. It is clear from the experimental data points that the response is lower for small angles and then reaches a relatively constant value for large angles. It is not unlikely that flat gas ionization chambers may show a similar but smaller angular dependence.

One probable explanation for the shape of the angular dependence of a liquid ionization chamber is the increased recombination when the electron tracks are parallel to the field lines, so that the electron and ion pairs that are produced have a larger probability to recombine before they reach the collector (7, 14).

Another possible reason for the angular dependence both for the diode and the ionization chamber could be interface phenomena that are known to depend on the direction of the beam (1, 2).

In the following sections the correction of the response of a detector in electron and photon beams is treated based on a measurement of the angular dependence of the detector.

Theory. The angular distribution of the electron fluence, Φ_Θ, in electron and photon beams is quite well described by a Gaussian angular distribution (4, 11, 12):

$$\Phi_\Theta(z) = \frac{e^{-\Theta^2/\overline{\Theta^2}(z)}}{\pi\overline{\Theta^2}(z)} \tag{1}$$

where $\overline{\Theta^2}(z)$ is the mean square angular spread at a depth z in the beam.

For electron beams $\overline{\Theta^2}(z)$ first increases with depth at a rate proportional to the linear scattering power (4) to reach a constant value of about 0.65 rad^2 beyond dose maximum (16). In photon beams the surface value is that due to the initial angular distribution of emitted compton and pair electrons

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and then reaches quickly a somewhat lower saturation value than in electron beams (Table) (13).

As can be seen from the solid and dashed lines in the Figure the angular dependence of the detector response $R(\Theta)$ normalized to unify at $\Theta=0$ can often be fitted by a function with a Gaussian component.

$$R(\Theta) = 1+\varepsilon_d-\varepsilon_d e^{-\Theta^2/\Theta_d^2} \qquad (2)$$

where ε_d is the relative variation of the detector response from small to large angles and Θ_d is a characteristic angle of the detector which describes how quickly the response varies with the angle Θ. A small Θ_d implies that the response increases rapidly for small angles up to around Θ_d and reaches its saturation value for $\Theta>1.5\ \Theta_d$.

However, when the angular dependence is determined experimentally the beam penetrating the detector will generally have a certain angular spread which implies that the measured detector response differs from the true response function for a monodirectional beam given by eq. 2. The measured detector response can be derived by using the distribution function of the projected angle Θ_x on the x, z plane ($\Theta^2=\Theta_x^2+\Theta_y^2$) (cf. 4):

$$R_m(\Theta) = \int_{-\infty}^{\infty}\left(1+\varepsilon_d-\varepsilon_d e^{-\Theta_x^2/\Theta_d^2}\right)\frac{e^{-(\Theta_x-\Theta)^2/\overline{\Theta^2}}}{\sqrt{\pi\overline{\Theta^2}}}\,d\Theta_x$$

$$= 1+\varepsilon_d\frac{\Theta_d^2}{\Theta_d^2+\overline{\Theta^2}}e^{-\frac{\Theta^2}{\Theta_d^2+\overline{\Theta^2}}} \qquad (3)$$

The measured response can thus be written:

$$R_m(\Theta) = k\left(1+\varepsilon_m-\varepsilon_m e^{-\Theta^2/\Theta_m^2}\right) \qquad (4)$$

where Θ_d^2, ε_d and k can now be determined from the measured quantities Θ_m^2 and ε_m according to:

$$\Theta_d^2 = \Theta_m^2-\overline{\Theta^2} \qquad (5)$$

$$\varepsilon_d = \varepsilon_m\frac{\Theta_d^2+\overline{\Theta^2}}{\Theta_d^2-\varepsilon_m\overline{\Theta^2}} \qquad (6)$$

and

$$k = 1+\varepsilon_d\frac{\overline{\Theta^2}}{\overline{\Theta^2}+\Theta_d^2}$$

From eq. 5 it is clearly seen that the measured distribution is wider than the intrinsic distribution of the detector. Since ε_m normally is much smaller than unity eq. 6 showes that the measured relative variation of the detector response is smaller than the true one which would be measured in a monodirectional beam (cf. also 3).

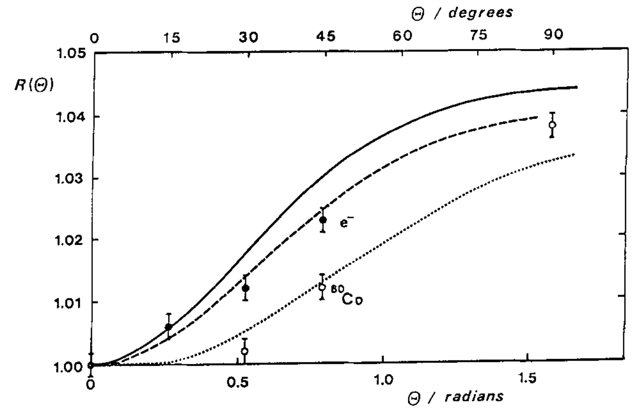

Fig. 1. Comparison of experimental and theoretical angular response functions R (Θ). Based on the response function measured in an 18 MeV electron beam (●---) the true response function for a monodirectional beam was calculated (——). From this curve the response in a ^{60}Co beam was calculated (. . . .) and compared with experiments (O).

If it is now assumed that the measured angular response function has been corrected according to eqs 5 and 6 so it is valid for beams with arbitrary angular distributions the detector response for a given angular distribution can be calculated according to:

$$R(z) = \int_0^{2\pi} R(\Theta)\,\Phi_\Theta(z)\,2\pi\sin\Theta d\Theta \qquad (7)$$

This integral is easily evaluated if the small angle approximation is used (replacing $\sin\Theta$ by Θ and 2π by ∞ as upper limit of integration) and the differential response function $R(\Theta)$ is taken from eq. 2 and the angular distribution from eq. 1. The depth dependence of the detector response then becomes simply:

$$R(z) = 1+\varepsilon_d\frac{\overline{\Theta^2}(z)}{\Theta_d^2+\overline{\Theta^2}(z)} \qquad (8)$$

The wider the angular spread in the beam, $\overline{\Theta^2}(z)$, the greater the detector response for a given Θ_d as should be expected for a positive ε_d. Eq. 8 gives a simple method to correct dose distributions measured with a detector of known angular dependence.

The correction is most important for electron beam depth dose curves where $\overline{\Theta^2}(z)$ varies continuously over most of the electron range but also in

Table

Mean square angular spread (rad²) in photon beams

Photon energy	$\overline{\Theta^2_{\sigma n}}/f_n$	$\overline{\Theta^2_{\sigma w}}/f_w$	$\overline{\Theta^2_{\varkappa}}/f_{\varkappa}$	$\overline{\Theta^2_i}/f_{21MV}$	$\overline{\Theta^2_{eq}}$
1.25 MeV	0.063/31	0.65/69	–	0.47/17	0.59
5 MeV	0.010/34	0.12/51	0.028/15	0.070/64	0.44
14 MeV	0.0019/16	0.044/33	0.0035/51	0.017/19	0.32
21 MV	0.020/30	0.24/51	0.005/19	0.13/100	0.43

the build-up region of a photon beam, where the mean square scattering angle again varies substantially as discussed in more detail below.

It should also be pointed out here that other shapes of the angular response function than given by eq. 2 may result in analytical solutions to eqs 3 and 7. This is for example the case for polynomials even though the mathematics may become more complex.

$\overline{\Theta^2}(z)$ *in photon beams.* Due to the lack of published data on the mean square scattering angle $\overline{\Theta^2}$ of the electrons in photon beams a table has been prepared for a few monoenergetic photon beam energies and a bremsstrahlung spectrum. In the Table the mean square scattering angle of the electrons at different photon beam energies in water are given. The last column gives the saturation value for the total slowing-down spectrum and pertains to depths from dose maximum or more exactly transient equilibrium and beyond. The first three columns give the initial contribution from the inelastic scattering ($\overline{\Theta^2_\sigma}$) and the pair production ($\overline{\Theta^2_\varkappa}$) processes. The former has been divided in a narrow and a wide angle group $\overline{\Theta^2_{\sigma n}}$ and $\overline{\Theta^2_{\sigma w}}$ with different mean electron energies to give an accurate fit to the angular dependence of the inelastic scattering cross-section (13). The corresponding percentage weight factors f_n, f_w, $f_\varkappa$ express the dose weighted contributions to the total mean square scattering angle. The factors f_{21MV} are corresponding percentage contributions to the bremsstrahlung spectrum.

The intrinsic mean square scattering angle, $\overline{\Theta^2_i}$, is the mean value over all initially generated electrons and positrons (photoelectric interactions have been disregarded). The equilibrium mean square scattering angle, $\overline{\Theta^2_{eq}}$, is the dose-weighted mean value over the complete slowing-down spectrum of the electrons. By comparing the $\overline{\Theta^2_i}$ and $\overline{\Theta^2_{eq}}$ values in this table it is seen that particularly at high energies, substantial variations in the angular distribution of electrons are obtained in the build-up region of a photon beam.

Comparison with experimental results. The experimental points in the Figure are obtained using a high energy electron beam ($\overline{E}_0 = 18.8$ MeV at a depth of 2 cm in water) and a ^{60}Co photon beam (at 10 cm depth in water). The corresponding mean square angular spreads $\overline{\Theta^2}(z)$ were 0.10 rad² and 0.59 rad², respectively (see above and the Table). Based on these theoretical values and on the fitting of the measured distribution (ε_m=0.04; Θ_m=0.80 rad; solid experimental points and the dashed curve in the Figure) the detector response for a monodirectional electron beam (solid line) and for a ^{60}Co beam at 10 cm depth (dotted line) was calculated according to the present theory.

Fairly good agreement is found both with regard to the fitting of the electron beam response and for the extrapolation to the response for a ^{60}Co beam. The half per cent discrepancy at 90° for the ^{60}Co response could partly be due to differential loss of photon attenuation in the low density liquid layer of the detector ϱ=0.67 g/cm² at this angle, but also due to the possibility that the interface effects in the chamber for photon and electron beams are somewhat different.

Discussion

With a thin plane-parallel liquid ionization chamber the dose in the surface region of an electron or photon beam may be underestimated by a few per cent due to the angular dependence of the detector. This may be circumvented by using a chamber with spherical electrodes rather than flat electrodes (17, 18) but an equally useful method is to make a correction according to the equations given in the present report. For some detectors such as semiconductor diodes the angular dependence can be quite large, 10 to 15 per cent is not uncommon, resulting in an increased distortion of the shape of the depth dose curve in both photon and electron beams (5).

The correction for a liquid chamber always amounts to less than ε_d, and is thus less than a few per cent in most cases. However, it is also possible to design silicon diodes with less than one per cent angular dependence by making the diameter of the silicon chip quite small (≲0.55 mm) and by drifting the PN junction into the silicon so a silicon detector with silicon equivalent walls is obtained. This will reduce the differential absorption effect and the interphase phenomena to the surrounding materials (9).

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