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SPECTRAL MEASUREMENTS AND MONTE CARLO CALCULATIONS OF SCATTERED RADIATION FROM THERAPEUTIC RADIATION SOURCES

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Generally, it has been assumed that the variation of the radiation quality due to scattering with increasing depth in a medium is small for photon radiation. Investigations (BRUCE et coll. 1963) based upon ^{60}Co have proved this assumption. For other radiation sources of interest, especially roentgen rays from linear accelerators, such investigations have been scarce. It is the purpose of the present report to give the results both of experiments and of calculations of the photon spectral distribution at various depths for 6 MeV roentgen rays and ^{60}Co gamma rays both used as therapeutic radiation sources.

The spectral measurements have been made on scattered radiation in a perspex phantom placed in a beam of gamma radiation from a T.E.M. Stabilatron kilocurie ^{60}Co therapy unit, and in a beam of roentgen radiation from a Varian Clinac 6 MeV linear accelerator. The measurements are performed with a NaI spectrometer. It was possible to change both the depth and the angle for analysis of the scattered radiation in the phantom.

For each depth the spectral distributions of the scattered radiation at the different angles are integrated to give the spectral distribution of the total fluence of scattered

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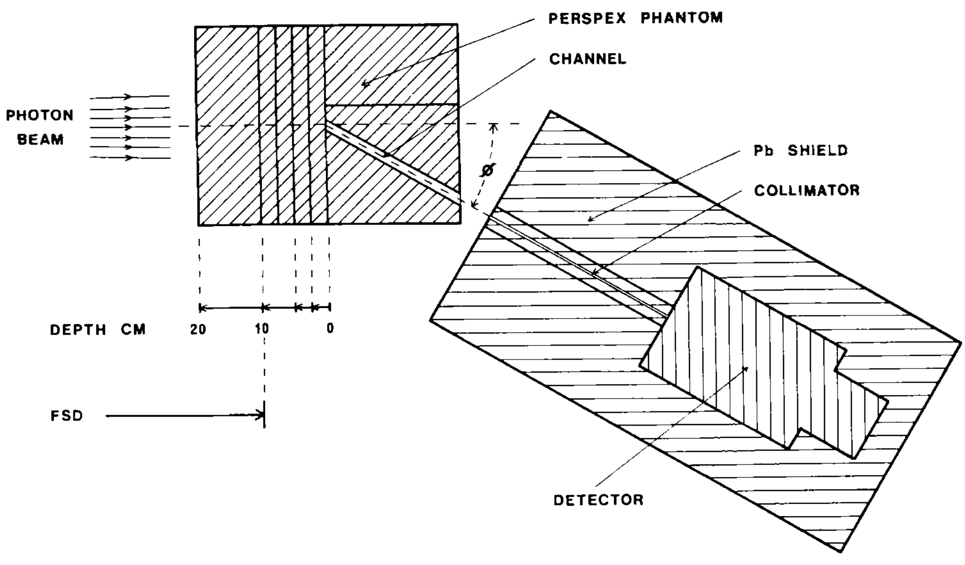

Fig. 1. Experimental arrangement.

photons. The integrated distributions are compared with Monte Carlo calculations. The computer program uses the measured primary spectra (JESSEN 1973) as input data and calculates the spectrum of scattered photons in the point of interest together with scaling-factors for the angular distribution. Although the number of photons in the calculated spectra is much lower than in the resulting measured spectra, the calculations are an essential confirmation of the integration of the contributions from the various scattering angles.

The integrated spectral distributions together with the distributions of the primary radiation for the same depths are used to calculate the spectra of the initial electrons released by the photon interaction processes, photoelectric effect, Compton effect and pair production. These electron spectra are applied for a calculation of the distributions of absorbed dose, of track length and of their average values, with respect to LET. Variations in the radiation qualities for the different depths in the phantom are examined in this way for the two different radiation sources.

Experimental procedure and data handling

The experimental arrangement is illustrated in Fig. 1. The phantom was a 30 cm $\times$ 30 cm $\times (20+d)$ cm cubic block of commercial, attainable perspex (mass density 1.19 g/cm^3 and electron density $3.833 \cdot 10^{23}$ el/cm^3). d is the depth varying from 2.5 cm to 20 cm. The scattering angle is defined by an air channel 1.5 cm in diameter. It is possible to change the scattering angle between seven values ranging from 7.5°

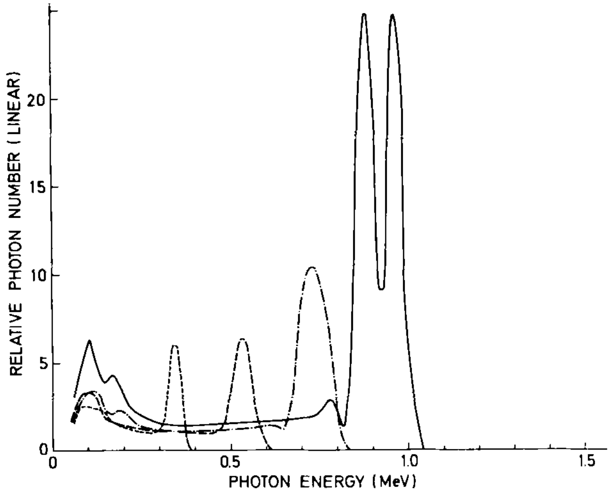

Fig. 2. Iterated spectral distributions from the ^{60}Co unit. Field size 4 cm × 4 cm. Depth 10 cm. Scattering angles: 30 deg. (——), 45 deg. (.–.–), 60 deg. (— —) and 90 deg. (- - -).

to 135° by replacing a perspex plate containing the channel. The phantom was placed in front of the spectrometer with facilities for fine alignement.

The spectrometer has a NaI(Tl) detector (diameter 12.7 cm) as described previously (JESSEN 1973). The cylindrical collimator, 30 cm in length, has a diameter of 3 or 5 mm depending on the fluence rate of scattered photons. The phantom was placed in the same measuring geometry as the scattering foil when measuring primary spectra. The SSD for the ^{60}Co source was 80 cm and the FSD for the measurements on the linear accelerator 685 cm. In order to reduce the background, measurements with the linear accelerator were performed with the phantom and the spectrometer placed behind a 1.6 m thick concrete wall supplied with a cylindric channel (6 cm in diameter), resulting in a field diameter on the phantom surface of 7 cm.

The pulse height distributions were measured by passing the output from the detector directly through a double delay-line amplifier into a multichannel analyser. A response-matrix for the detector was used to correct the pulse height distributions for effects from the crystal and the following electronics, after subtraction of the spectrum of background radiation measured with the phantom removed from the radiation beam. Mathematically, it may be expressed by the matrix equation $P = R \cdot N$, where P is measured pulse height distribution, R the response-matrix and N the scattered photon spectrum. An iterative procedure was applied for solving this complicated equation (SKARSGÅRD et coll. 1961).

The iterated spectral distributions at four different measuring angles on the ^{60}Co unit for a field size of 4 cm × 4 cm and in a phantom depth of 10 cm are given in Fig. 2. The distributions are normalized to the same absorbed dose. The figure illustrates the preferred forward scattering and the good resolution of the two peaks up to 30 degrees scattering angle.

The spectral distributions of the scattered radiation in the different measuring

Table 1

The ratio between the total measured and calculated photon numbers for various scattering angles and collimator diameters (relative to the ratio at 45°)

Φ deg.	Stabilatron coll. 3 or 5 mm	Accelerator	
		coll. 5 mm	coll. 3 mm
7.5	1.09	—	—
15	1.07	—	0.98
30	1.00	0.67	1.07
45	1	1	1
60	0.94	1.02	—
90	1.04	1.00	—

angles for the same field and depth were integrated over all angles Φ. This was carried out after inter- and extrapolation between the measured values to Φ intervals of 15 degrees for the roentgen rays and 3 degrees for the ^{60}Co gamma rays, respectively. The system is radially symmetrical about the central axis of the primary beam and integration over the azumithal angle may therefore be performed by multiplying by $2\pi \sin \Phi$. Integration of all scattering angles was then done by numerical integration after Simpson's rule of the function $2\pi f(\varepsilon, \Phi) \sin \Phi$ for the center value ε of each energy interval. $f(\varepsilon, \Phi)$ is the photon number at the scattering angle Φ and with energy ε.

Monte Carlo calculations

The solution of transport problems for photon radiation by means of Monte Carlo calculations is a wellknown method (BRUCE & JOHNS 1960, ZERBY 1963) which may be applied successfully if a large computer is available. The main problem is the sampling from the Klein-Nishina formula and in the present calculations the method given by Kahn was applied (ZERBY 1963).

The program has been written to simulate the experimental setup as much as possible: the starting energy and number of photons were defined by the measured primary spectra and each photon was followed individually through a 30 cm × 30 cm × 30 cm perspex phantom, from when it entered through a 4 cm × 4 cm rectangular field for the ^{60}Co source or a 7 cm in diameter circular field for the linear accelerator until it escaped from the phantom or was absorbed within it.

Photons which cross a small sphere around the centerpoints in the depths 3.5, 10 and 20 cm (only 10 cm for the Co source) are recorded by energy and angle and the result of the calculation is presented as the spectrum of scattered photons in the different depths and the total number of photons scattered into the measuring angles specified above.

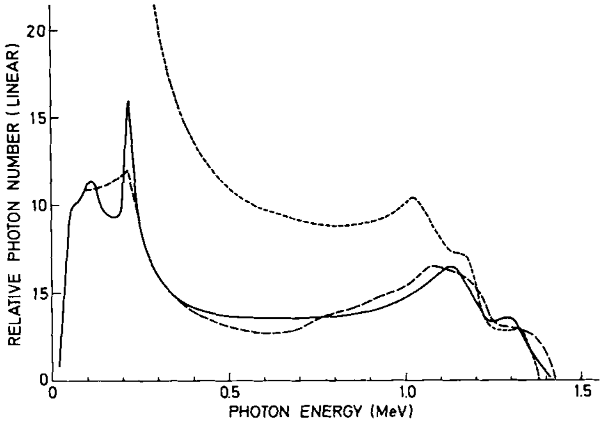

Fig. 3. Integrated distributions of the scattered photons from the ^{60}Co unit. Depth 10 cm. Field size 4 × 4 cm: Monte Carlo calculations (——) and measured (— —). Field size 15 cm × 15 cm: measured (- - -).

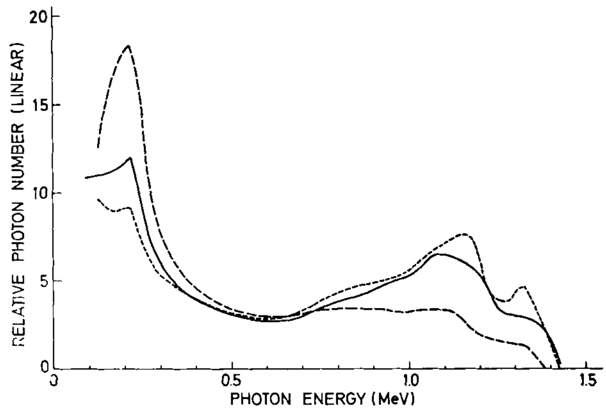

Fig. 4. Integrated distributions of the scattered photons from the ^{60}Co unit. Field size 4 cm × 4 cm. Depths: 2.5 cm (— —), 10 cm (——) and 20 cm (- - -).

The values of the interacting coefficients for perspex are taken from NBS 29 (Hubbell 1969) and the following simplifications have been made: below 10 keV or 25 keV for the Co source and the linear accelerator respectively all photons are absorbed immediately; between 150 and 1 500 keV only compton scattering is effective, and above 1 500 keV a small fraction of the photons undergoes pair production. If a photon is absorbed by pair production one 511 keV photon is sent out in an arbitrary direction from the point of interaction. For the linear accelerator 0.3 per cent of all the interactions were pair productions. In this calculation it is computer-time saving to express the energy in keV as an integer variable which may be used to call attenuation coefficients etc. from numerical arrays.

The calculations have been carried out in several steps, and it is therefore possible to estimate the coefficient of variation of the number of photons given by the calculations to be below 5 per cent in most cases.

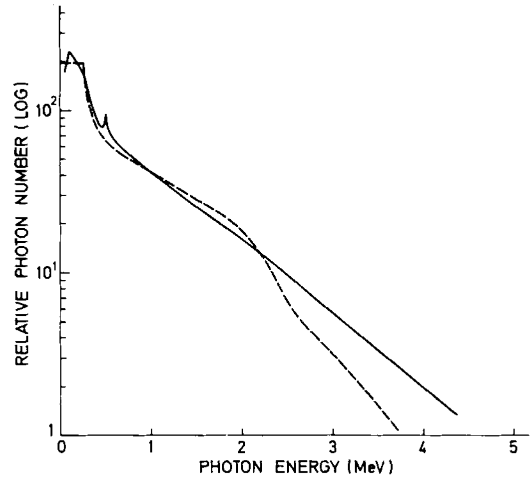

Fig. 5

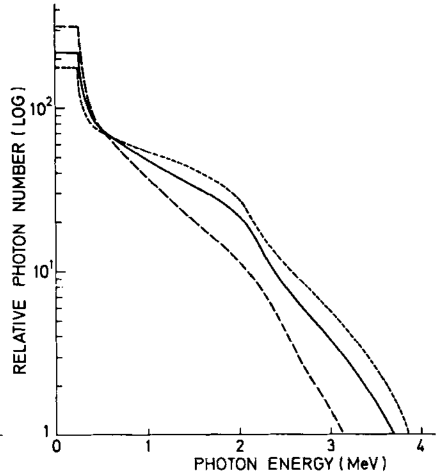

Fig. 6

Fig. 5. Integrated distributions of the scattered photons from the 6 MeV roentgen rays. Depth 10 cm. Monte Carlo calculations (——) and measured (---).

Fig. 6. Integrated distributions of the scattered photons from the 6 MeV roentgen rays. Depths: 3.5 cm (— —), 10 cm (——) and 20 cm (---).

Results and Comparisons

The ratio r between the total measured and calculated photon numbers for the scattering angle 45 degrees is about 10^6. The ratio relative to this number for the various scattering angles appears in Table 1. The distributions for the different angles were normalized to the same absorbed dose for the incoming radiation. For the ^{60}Co source no dependence on collimator size was observed and all measurements can be carried out with both the 3 mm and the 5 mm collimators at all angles. For the linear accelerator the fluence rate is too high at 15 and 30 degrees for the 5 mm collimator to be used, and only the 3 mm collimator was applied at these angles. This probably means a saturation of the analyser system caused by the great number of forward scattered photons sent towards the solid angle of the detector at high energies. The analyser cannot analyse more than one photon per radiation pulse from the accelerator.

In Fig. 3 the Monte Carlo calculations are compared with the measured distribution of scattered photons for the 4 cm $\times$ 4 cm field in a phantom depth of 10 cm for ^{60}Co gamma radiation. The two curves are normalized to the same total photon number. Although there are some differences in the resolutions around the backscatter peak about 215 keV, there seems to be a good agreement between the calculated and measured spectra, which verified the method used for integration of the measured distributions. From this agreement for the 4 cm $\times$ 4 cm field it is concluded that the measured distributions for other fields and depths are valuable representations. In

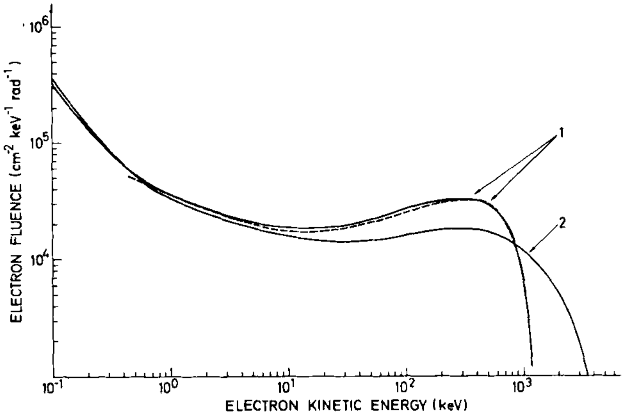

Fig. 7. Electron fluence distributions of secondary and slowing down electrons for (1) ^{60}Co gamma radiation and (2) 6 MeV roentgen rays. Depth 10 cm. ---- from ICRU 16.

Fig. 3 is also plotted a curve for the 15 cm × 15 cm field at the same depth and normalized to the same absorbed dose as for the 4 cm × 4 cm field.

The integrated distributions of the measured scattered photons in three depths of the phantom are drawn in Fig. 4 for the 4 cm × 4 cm fields again normalized to the same total photon number. The figure illustrates a filtration with depth, i.e. the relative content of the high energy photons in the scattered spectrum is increased for increasing depth in the phantom material.

For the pulsed 6 MeV roentgen rays from the accelerator the analogous comparisons were performed and the distributions for the calculated and measured scattered photons at a depth of 10 cm appear in Fig. 5. Only a circular radiation field of 7 cm in diameter was used for the accelerator experiments. The 511 keV annihilation peak in the Monte Carlo calculations, which could be caused by the model used, is not seen in the measured distribution. Probably also the response matrix cleans the pulse height distributions too strongly at this energy. The deviation for the higher energies could be expected in this semilog plot, as the lowest measuring angle used for the integration is 15 degrees.

In Fig. 6 the integrated distributions of the measured scattered photons for three depths are plotted. The distributions are also here normalized to the same total photon number, and illustrate, as for the distributions for ^{60}Co, a filtration with depth.

LET calculations. The primary photon distributions measured previously (JESSEN 1973) are added to the scattered distributions to obtain the total photon fluence for LET calculations. The primary spectra were corrected to the measuring depth for filtration in the phantom material and were normalized to the same absorbed dose as represented by the scattered distributions.

These total photon distributions were used to calculate the spectra of the initial electrons set in motion per gram of the perspex material at an absorbed dose of 1

Table 2

The absorbed dose average LET, $\bar{L}_{100,D}$, and the track average LET, $\bar{L}_{100,T}$, for the ^{60}Cogamma rays for different field sizes and depths in the perspex phantom. The average values are in keV/μm

Depth cm	4 cm × 4 cm		15 cm × 15 cm	
	$\bar{L}_{100,D}$	$\bar{L}_{100,T}$	$\bar{L}_{100,D}$	$\bar{L}_{100,T}$
2.5	6.72	0.289	6.68	0.295
5	6.72	0.288	6.68	0.294
10	6.72	0.287	6.67	0.293
20	6.73	0.285	6.65	0.293

rad. The lower energy cut-off value for the photon spectra was 100 keV set by the analyser system. Cross sections for the photoelectric effect were calculated after NBS 29 (Hubbell 1969), and the mean excitation-ionization potential (I = 65.7 eV) for defining the energy distributions of the photoelectrons is given by Bragg's rule (ICRU 16, 1970). Cross sections and energy distributions of the electrons formed by Compton and pair production processes were calculated after formulas given by Bethe & Ashkin (1953).

From the normalized initial electron spectra the electron slowing down spectra were calculated by means of McGinnies' tables (NBS 597, 1959) for water. The values are extrapolated graphically from 451.7 eV down to 100 eV, the cut-off energy for the LET calculations, although the limit is exceeded for the Born approximation, as the binding energies for the K electrons of the oxygen and the carbon are 532 eV and 284 eV respectively. But the binding energies for the rest (74 per cent) of the electrons in the molecules are below 100 eV and therefore it was assumed that the Born approximation still was valid. The extrapolation of the slowed-down and secondary electrons for a certain initial electron energy is in agreement with the results from Bruce et coll. (1963), where a similar assumption was made. The small differences in Z/A and I between water and perspex are neglected.

Fig. 7 illustrates the calculated energy spectrum resulting from slowing down and secondary electrons produced by (1) ^{60}Co gamma radiation and (2) 6 MeV roentgen rays at a depth of 10 cm. There seems to be a good agreement with the spectrum for ^{60}Co taken from ICRU 16.

The slowing down spectra were used for calculations, for all depths and field sizes measured, of the track length distributions, the absorbed dose distributions and their averages with respect to LET. The energy cut-off in these calculations were 100 eV. The restricted electron stopping power for 0.1 to 10 keV electrons was taken from table A1 in ICRU 16 and for the kinetic energies above 10 keV, the restricted stopping power was calculated after Rohrlich & Carlson (1964). The density correction was performed after Sternheimer (1956). Fig. 8 illustrates the cumulative absorbed dose

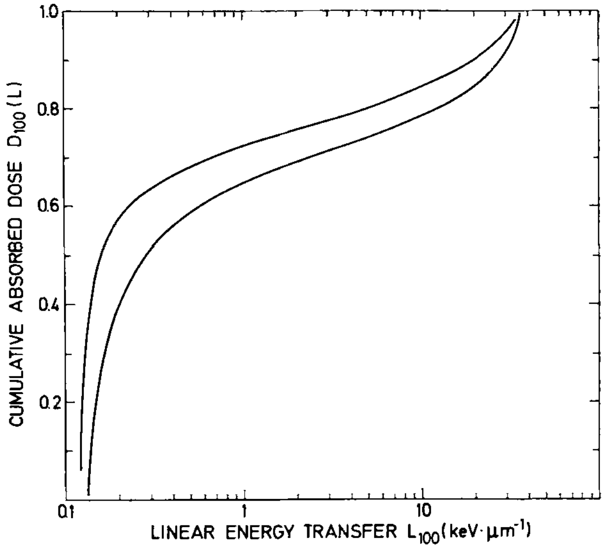

Fig. 8. Cumulative absorbed dose distributions in LET for ^{60}Co gamma radiation (lower curve) and for 6 MeV roentgen rays (upper curve). Depth 10 cm.

distributions in LET for the 4 cm × 4 cm field of ^{60}Co gamma rays and for the 6 MeV roentgen rays at a depth of 10 cm. Sixty per cent of the absorbed dose are deposited with LET values below 0.58 keV/μm for ^{60}Co and below 0.23 keV/μm for the 6 MeV roentgen rays.

In Table 2 the average values in LET for the ^{60}Co source are given for different fields and depths. There seems to be only a little spread in these data which supports the assumption that there should be a very small variation in the radiation quality with increasing depth. There is a good agreement with the dose average with respect to LET given by ICRU 16 (6.9 keV/μm) for cut-off energy at 100 eV, but the track average is given a little lower by ICRU (0.229 keV/μm). Otherwise the agreement is

Table 3

The absorbed dose average LET, $\bar{L}_{100,D}$, and the track average LET, $\bar{L}_{100,T}$, for the 6 MeV roentgen rays different depths in the phantom. The average values are in keV/µm

Depth cm	Diam. 7 cm $\bar{L}_{100,D}$	Diam. 7 cm $\bar{L}_{100,T}$
3.5	6.97	0.243
10	6.96	0.239
20	6.97	0.235

reversed with the data given by Bruce et coll. ($\bar{L}_D = 8.3$ keV/μm and $\bar{L}_T = 0.32$ keV/μm). Table 3 gives the average values in LET for the 6 MeV roentgen rays. The results indicate small variations with depth similar to the results for the ^{60}Co source, but suggest a deviation in average values for the two radiation sources. The calculated averages in LET are very sensitive to the stopping power for the low energy electrons, but the calculations are based upon the same stopping power for these energies indicating that there should only be a small difference in biologic effect for the two radiation sources.

Acknowledgement

The data handling and the LET calculations were performed on the CDC-6400 computer at the University of Aarhus and for the Monte Carlo calculations the IBM 370/165 computer at NEUCC, Copenhagen was used. The computer programs are written in Fortran IV.

SUMMARY

Spectral measurements have been made on scattered radiation in a perspex phantom in various depths from a kilocurie ^{60}Co unit and a 6 MeV linear accelerator using a NaI crystal spectrometer. The measurements are compared to Monte Carlo calculations and a good agreement is obtained. The measured total photon distributions make the basis for calculations of absorbed dose and track length distributions in LET and their average values for the different depths and fields.

ZUSAMMENFASSUNG

Es wurden spektrale Messungen der Streustrahlung unter Verwendung eines NaJ Kristall-Spektrometers an einem Perspex-Phantom in verschiedenen Abständen von einer Kilokuri ^{60}Co-Einheit und einem 6 MeV Linearaccelerator vorgenommen. Die Messungen wurden mit Monte Carlo Berechnungen verglichen wobei eine gute Übereinstimmung erhalten wurde. Die gemessene Gesamt-Photonen-Verteilung bildet die Basis für Berechnungen der absorbierten Dosis und der Spuren-Längen-Verteilungen der LET und deren Durchschnittswerte für verschiedene Tiefen und Felder.

RÉSUMÉ

Des mesures spectrales ont été faites au moyen d'un spectromètre à cristal de NaI sur le rayonnement diffusé dans un fantôme de perspex à différentes profondeurs recevant le rayonnement d'une source d'un kilocurie de ^{60}Co et d'un accélérateur linéaire de 6 MeV. Ces mesures ont été comparées aux calculs de Monte Carlo, et les auteurs ont constaté une bonne concordance. Les distributions totales de photons mesurées ont servi de base pour les calculs de dose absorbée et pour les distributions de longueur de trajectoire en LET et pour leur valeur moyenne pour les différentes profondeurs et les différents champs.

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